

# C3 FUNCTIONS

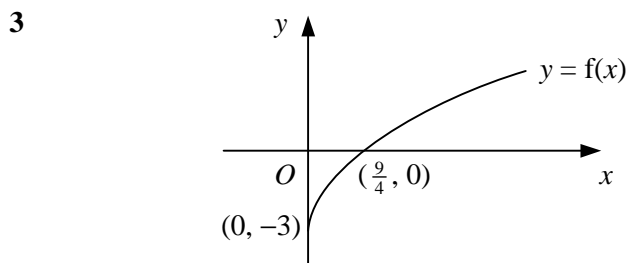
## Worksheet G

- 1  $f: x \rightarrow 2 + \log_4 x, x \in \mathbb{R}, x > 0.$
- a Evaluate  $ff(1).$  (3)
- b Solve the equation  $f(x) = 0.$  (2)
- c Find the inverse function  $f^{-1}(x).$  (3)

- 2 The function  $f$  is defined by
- $$f: x \rightarrow |3x - a|, x \in \mathbb{R}.$$

where  $a$  is a positive constant.

- a Find  $ff(-2a).$  (2)
- b Sketch the graph  $y = f(x)$ , showing the coordinates of any points where the graph meets the coordinate axes. (3)
- c Solve the equation  $f(x) = x$ , giving your answers in terms of  $a.$  (3)



The diagram shows the graph of  $y = f(x)$  which meets the  $x$ -axis at the point  $(\frac{9}{4}, 0)$  and the  $y$ -axis at the point  $(0, -3).$

- a Sketch on separate diagrams the graphs of
- i  $y = |f(x)|,$
- ii  $y = f^{-1}(x).$  (4)
- Given that  $f(x)$  is of the form  $f(x) \equiv ax^{\frac{1}{2}} + b, x \in \mathbb{R}, x \geq 0,$
- b find the values of the constants  $a$  and  $b,$  (3)
- c find an expression for  $f^{-1}(x).$  (3)

- 4 The function  $f$  is defined by
- $$f: x \rightarrow \frac{x+2}{x-1}, x \in \mathbb{R}, x \neq 1.$$
- a Show that  $ff(x) = x$  for all  $x \in \mathbb{R}, x \neq 1.$  (3)
- b Hence, write down an expression for  $f^{-1}(x).$  (1)

The function  $g$  is defined by

$$g: x \rightarrow 2x - 3, x \in \mathbb{R}.$$

- c Solve the equation  $gf(x) = 0.$  (4)
- 5 a Sketch on the same set of axes the graphs of  $y = |x|$  and  $y = |2x - 3|.$  (3)
- b Hence, or otherwise, solve the equation
- $$|x| = |2x - 3|. \quad (4)$$

**C3 FUNCTIONS***Worksheet G continued*

- 6** The function  $f(x)$  is defined for all real values of  $x$  by

$$f(x) = x + 2, \quad x < 1,$$

$$f(x) = 4 - x^2, \quad x \geq 1.$$

- a** Sketch the graph of  $f(x)$  showing the coordinates of any points of intersection with the coordinate axes. (4)
- b** Evaluate  $ff(3)$ . (2)
- c** Solve the equation  $f(x) = 1$ . (4)

- 7** The functions  $f$  and  $g$  are defined by

$$f : x \rightarrow kx + 2, \quad x \in \mathbb{R},$$

$$g : x \rightarrow x - 3k, \quad x \in \mathbb{R},$$

where  $k$  is a constant.

- a** Find expressions in terms of  $k$  for
- i**  $f^{-1}(x)$ ,
- ii**  $fg(x)$ . (4)

Given that  $fg(7) = 4$ ,

- b** find the two possible values of  $k$ . (3)
- 8**  $f(x) \equiv x^2 - 4x + 5, \quad x \in \mathbb{R}, \quad x \geq 2$ .
- a** Express  $f(x)$  in the form  $a(x + b)^2 + c$ . (2)
- b** State the range of  $f$ . (1)
- c** Find an expression for  $f^{-1}(x)$  and state its domain. (4)
- d** Sketch the graphs of  $y = f(x)$  and  $y = f^{-1}(x)$  on the same diagram and state the relationship between the graphs. (4)

- 9** The functions  $f$  and  $g$  are defined by

$$f : x \rightarrow x^2 + 4, \quad x \in \mathbb{R},$$

$$g : x \rightarrow 2x - \frac{1}{x}, \quad x \in \mathbb{R}, \quad x \neq 0.$$

- a** Evaluate  $gf(-2)$ . (2)
- b** Find and simplify an expression for  $fg(x)$ . (3)
- c** Find the values of  $x$  for which  $fg(x) = 5$ . (4)

- 10** The function  $f$  is given by

$$f : x \rightarrow e^{\frac{1}{2}x} - 3, \quad x \in \mathbb{R}.$$

- a** Find  $f^{-1}(x)$  and state its domain. (4)
- b** Sketch the curve  $y = f^{-1}(x)$ , showing the coordinates of any points of intersection with the coordinate axes. (3)

The function  $g$  is given by

$$g : x \rightarrow \ln(x + 5), \quad x \in \mathbb{R}, \quad x > -5.$$

- c** Evaluate  $fg(4)$ . (2)
- d** Solve the equation  $f^{-1}(x) = g(x)$ . (4)