

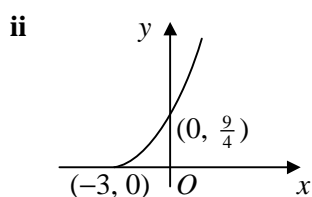
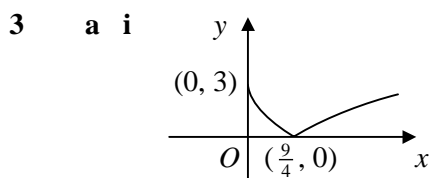
C3 FUNCTIONS

Answers - Worksheet G

1 a $f(1) = 2 + \log_4 1 = 2$
 $ff(1) = f(2) = 2 + \log_4 2$
 $= 2 + \frac{1}{2} = \frac{5}{2}$

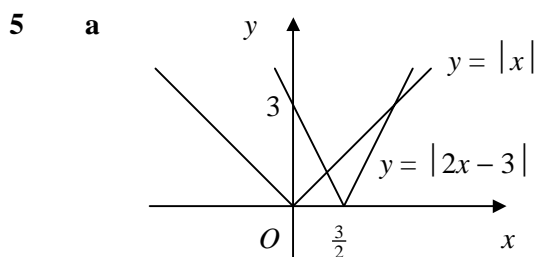
b $2 + \log_4 x = 0$
 $\log_4 x = -2$
 $x = 4^{-2} = \frac{1}{16}$

c $y = 2 + \log_4 x$
 swap $x = 2 + \log_4 y$
 $\log_4 y = x - 2$
 $y = 4^{x-2}$
 $f^{-1}(x) = 4^{x-2}, x \in \mathbb{R}$



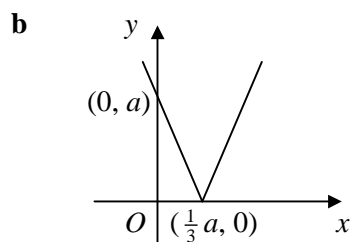
b $f(0) = b = -3$
 $f(\frac{9}{4}) = \frac{3}{2}a - 3 = 0$
 $\therefore a = 2, b = -3$

c $y = 2x^{\frac{1}{2}} - 3$
 swap $x = 2y^{\frac{1}{2}} - 3$
 $y^{\frac{1}{2}} = \frac{1}{2}(x+3)$
 $f^{-1}(x) = \frac{1}{4}(x+3)^2, x \in \mathbb{R}, x \geq -3$



b $x = 2x - 3 \Rightarrow x = 3$
 $x = -(2x - 3) \Rightarrow x = 1$
 $\therefore x = 1, 3$

2 a $f(-2a) = |-7a| = 7a$
 $ff(-2a) = f(7a) = |20a| = 20a$

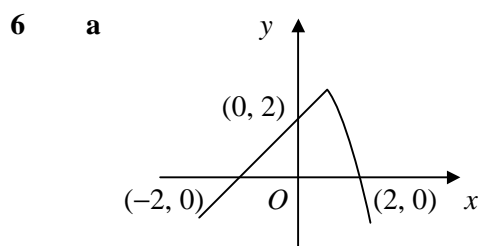


c $3x - a = x \Rightarrow x = \frac{1}{2}a$
 $-(3x - a) = x \Rightarrow x = \frac{1}{4}a$
 $\therefore x = \frac{1}{4}a, \frac{1}{2}a$

4 a $ff(x) = f(\frac{x+2}{x-1}) = \frac{\frac{x+2}{x-1}+2}{\frac{\frac{x+2}{x-1}-1}$
 $= \frac{x+2+2(x-1)}{x+2-(x-1)}$
 $= \frac{3x}{3} = x$

b $f^{-1}(x) = \frac{x+2}{x-1}, x \in \mathbb{R}, x \neq 1$

c $gf(x) = 2f(x) - 3 = 0$
 $\therefore f(x) = \frac{x+2}{x-1} = \frac{3}{2}$
 $2(x+2) = 3(x-1)$
 $2x+4 = 3x-3$
 $x = 7$



b $f(3) = 4 - 9 = -5$
 $ff(3) = f(-5) = -5 + 2 = -3$
 c $x + 2 = 1 \Rightarrow x = -1$
 $4 - x^2 = 1 \Rightarrow x^2 = 3$
 $x \geq 1 \therefore x = \sqrt{3}$
 $\therefore x = -1, \sqrt{3}$

7 a i $y = kx + 2$
 swap $x = ky + 2$
 $y = \frac{x-2}{k}$
 $f^{-1}(x) = \frac{x-2}{k}, x \in \mathbb{R}$

ii $fg(x) = f(x - 3k)$
 $= k(x - 3k) + 2$
 $fg(x) = kx - 3k^2 + 2, x \in \mathbb{R}$

b $4 = 7k - 3k^2 + 2$
 $3k^2 - 7k + 2 = 0$
 $(3k - 1)(k - 2) = 0$
 $k = \frac{1}{3}, 2$

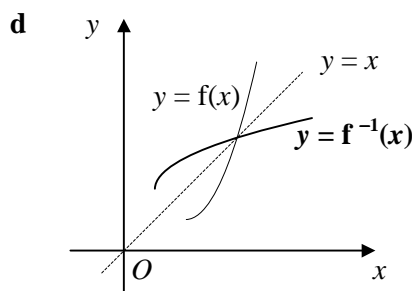
8 a $f(x) = (x - 2)^2 - 4 + 5 = (x - 2)^2 + 1$
 b $f(x) \geq 1$
 c $y = (x - 2)^2 + 1$

swap $x = (y - 2)^2 + 1$

$y = 2 \pm \sqrt{x - 1}$

(domain of $f \Rightarrow +$)

$f^{-1}(x) = 2 + \sqrt{x - 1}, x \in \mathbb{R}, x \geq 1$

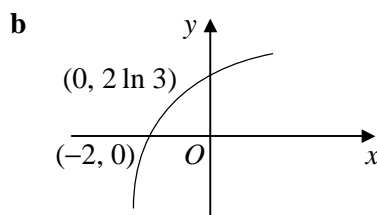


the graphs are reflections of each other
 in the line $y = x$

9 a $f(-2) = 8$
 $gf(-2) = g(8) = 15\frac{7}{8}$
 b $fg(x) = f(2x - \frac{1}{x}) = (2x - \frac{1}{x})^2 + 4$
 $= 4x^2 - 4 + \frac{1}{x^2} + 4$
 $fg(x) = 4x^2 + \frac{1}{x^2}$

c $4x^2 + \frac{1}{x^2} = 5$
 $4x^4 - 5x^2 + 1 = 0$
 $(4x^2 - 1)(x^2 - 1) = 0$
 $x^2 = \frac{1}{4}, 1$
 $x = \pm\frac{1}{2}, \pm 1$

10 a $y = e^{\frac{1}{2}x} - 3$
 swap $x = e^{\frac{1}{2}y} - 3$
 $\frac{1}{2}y = \ln(x + 3)$
 $y = 2 \ln(x + 3)$
 $f^{-1}(x) = 2 \ln(x + 3), x \in \mathbb{R}, x > -3$



c $g(4) = \ln 9 = 2 \ln 3$
 $fg(4) = f(2 \ln 3) = 0$
 d $2 \ln(x + 3) = \ln(x + 5)$
 $(x + 3)^2 = x + 5$
 $x^2 + 5x + 4 = 0$
 $(x + 4)(x + 1) = 0$
 $x > -3 \therefore x = -1$