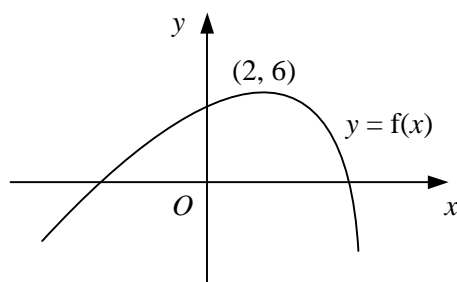


# C3 FUNCTIONS

## Worksheet E

- 1 Describe how the graph of  $y = f(x)$  is transformed to give the graph of
- a**  $y = 2 + f(x + 3)$       **b**  $y = 2f(-x)$       **c**  $y = 3f(x - 1)$       **d**  $y = 4 - f(x)$
- 2 **a** Express  $x^2 + 6x + 2$  in the form  $a(x + b)^2 + c$ .  
**b** Hence, describe two transformations that would map the graph of  $y = x^2$  onto the graph of  $y = x^2 + 6x + 2$ .
- 3 Each of the following graphs is translated by 3 units in the positive  $x$ -direction and then stretched by a factor of 2 in the  $y$ -direction, about the  $x$ -axis.  
 Find and simplify an equation of the graph obtained in each case.
- a**  $y = 2x + 7$       **b**  $y = 3e^x$       **c**  $y = x^2 - 3x + 1$       **d**  $y = \frac{1}{x}$
- 4 Describe in order two transformations that would map the graph of
- a**  $y = |x|$  onto the graph of  $y = -|3x|$       **b**  $y = e^x$  onto the graph of  $y = 5 + e^{-x}$   
**c**  $y = \frac{1}{x}$  onto the graph of  $y = \frac{3}{x+4}$       **d**  $y = \ln x$  onto the graph of  $y = 2 + 3 \ln x$

5



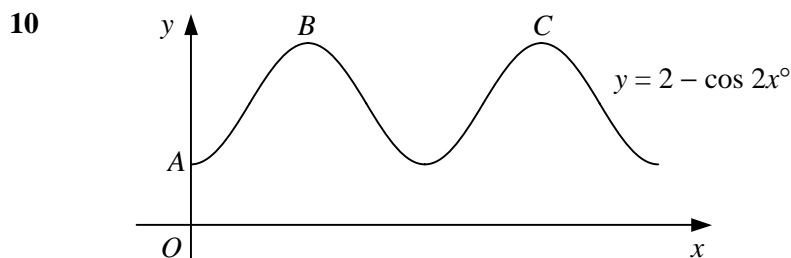
The diagram shows the curve with equation  $y = f(x)$  which is stationary at the point  $(2, 6)$ .  
 Showing the coordinates of the stationary point in each case, sketch on separate diagrams the graphs of

- a**  $y = 1 + f(x - 4)$       **b**  $y = 3 - f(x)$       **c**  $y = 2f(x + 1)$       **d**  $y = \frac{1}{2}f(2x)$
- 6 The graph of  $y = x^2 + 4x - 2$  undergoes the following three transformations:  
 first: translation by  $-2$  units in the positive  $x$ -direction,  
 second: stretch by a factor of 3 in the  $y$ -direction, about the  $x$ -axis,  
 third: reflection in the  $y$ -axis.  
 Find and simplify an equation of the graph obtained.
- 7 **a** Express  $2x^2 - 4x + 7$  in the form  $a(x + b)^2 + c$ .  
**b** Hence, describe in order a sequence of transformations that would map the graph of  $y = 2x^2 - 4x + 7$  onto the graph of  $y = x^2$ .
- 8  $f(x) \equiv x^3 - 3x^2 + 4$ ,  $x \in \mathbb{R}$ .
- a** Find the coordinates of the stationary points on the graph of  $y = f(x)$ .  
**b** Hence, find the coordinates of the stationary points on each of the following graphs.
- i**  $y = -2f(x)$       **ii**  $y = 3 + f(\frac{1}{2}x)$       **iii**  $y = \frac{1}{4}f(x - 2)$

## C3 FUNCTIONS

## Worksheet E continued

- 9 a Describe clearly, in order, the sequence of transformations that would map the graph of  $y = \sqrt{x}$  onto the graph of  $y = 2 - 3\sqrt{x}$ .
- b Sketch the graph of  $y = 2 - 3\sqrt{x}$  showing the coordinates of any points where the graph meets the coordinate axes.



The diagram shows part of the curve with equation  $y = 2 - \cos 2x^\circ$ ,  $x > 0$ .

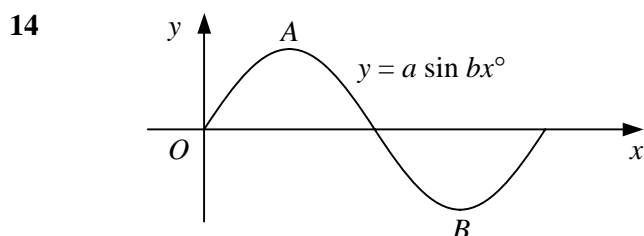
- a State the period of the curve.
- b Write down the coordinates of the point A where the curve meets the y-axis.
- c Write down the coordinates of B and C, the first two maximum points on the curve.
- 11 Sketch each of the following curves for  $x$  in the interval  $0 \leq x \leq 360$ . Show the coordinates of any turning points and the equations of any asymptotes.
- |                                     |                                   |                            |
|-------------------------------------|-----------------------------------|----------------------------|
| a $y = 3 \cos 2x^\circ$             | b $y = \tan (-2x^\circ)$          | c $y = 1 + 2 \sin x^\circ$ |
| d $y = -\sin (x + 60)^\circ$        | e $y = 2 \cos (x - 45)^\circ$     | f $y = 3 - \tan x^\circ$   |
| g $y = 2 + \cos \frac{1}{2}x^\circ$ | h $y = 4 \sin \frac{3}{2}x^\circ$ | i $y = 1 - 2 \cos x^\circ$ |

- 12 State the period of the curves with the equations

- a  $y = 2 \tan 3x^\circ$ ,
- b  $y = 1 + \sin kx^\circ$ , giving your answer in terms of  $k$ .

- 13  $f(x) \equiv 2 \sin \frac{1}{2}x$ ,  $0 \leq x \leq 2\pi$ .

- a Sketch the graph  $y = f(x)$ .
- b State the coordinates of the maximum point of the curve.
- c Solve the equation  $f(x) = \sqrt{2}$ , giving your answers in terms of  $\pi$ .



The graph shows the curve  $y = a \sin bx^\circ$ ,  $0 \leq x \leq 180$ .

The curve has a maximum at the point A with coordinates (45, 4).

- a Find the values of the constants  $a$  and  $b$ .
- b Write down the coordinates of the minimum point of the curve, B.