

C3

FUNCTIONS

Worksheet B

1 $f: x \rightarrow 4x - 3, x \in \mathbb{R}$ $g: x \rightarrow 2 - x, x \in \mathbb{R}$ $h: x \rightarrow x^2 + 5, x \in \mathbb{R}$

Evaluate

a $gf(2)$ **b** $gh(1)$ **c** $fg(-3)$ **d** $hf(3)$
e $gg(5)$ **f** $ff(\frac{1}{2})$ **g** $hg(8)$ **h** $fh(1\frac{1}{2})$

2 $f: x \rightarrow 5x + 2, x \in \mathbb{R}$ $g: x \rightarrow \cos x, x \in \mathbb{R}$ $h: x \rightarrow \ln x, x \in \mathbb{R}, x > 0$

Evaluate, giving your answers to 3 significant figures

a $fh(20)$ **b** $gh(3)$ **c** $fg(5)$ **d** $gg(-4)$
e $gf(1\frac{3}{4})$ **f** $hg(6.7)$ **g** $hh(50)$ **h** $hf(-0.3)$

3 $f: x \rightarrow 2x + 1, x \in \mathbb{R}$ $g: x \rightarrow 1 - 3x, x \in \mathbb{R}$ $h: x \rightarrow x^2 + 4, x \in \mathbb{R}$

Given the functions f , g and h , express the following composite functions in a similar form.

a fg **b** ff **c** fh **d** hf
e gh **f** gg **g** hg **h** gf

4 $f: x \rightarrow 4 - x, x \in \mathbb{R}$ $g: x \rightarrow e^x, x \in \mathbb{R}$ $h: x \rightarrow 2x^2 + 7, x \in \mathbb{R}$

Given the functions f , g and h , express the following composite functions in a similar form.

a gf **b** hg **c** fh **d** gg
e gh **f** ff **g** fg **h** hf

5 $f: x \rightarrow 5x - 3, x \in \mathbb{R}$ $g: x \rightarrow 3x^2 + 1, x \in \mathbb{R}$ $h: x \rightarrow \frac{1}{x-2}, x \in \mathbb{R}, x \neq 2$

Solve

a $ff(x) = -8$ **b** $hf(x) = 2$ **c** $gf(x) = 28$ **d** $hg(x) = \frac{1}{2}$
e $fh(x) = 7$ **f** $fg(x) = 32$ **g** $gh(x) = 4$ **h** $hh(x) = -2$

6 $f: x \rightarrow \ln x, x \in \mathbb{R}, x > 0$ $g: x \rightarrow 3 + 2x, x \in \mathbb{R}$ $h: x \rightarrow e^x, x \in \mathbb{R}$

Solve, giving your answers to 2 decimal places,

a $gh(x) = 9$ **b** $fg(x) = 3.6$ **c** $hg(x) = 4$ **d** $gf(x) = 10.4$

7 The functions f and g are defined by

$$f: x \rightarrow \frac{x+1}{5}, x \in \mathbb{R} \qquad g: x \rightarrow e^x, x \in \mathbb{R}$$

a State the range of g .

b Solve $fg(x) = 17$.

8 The functions f and g are defined by

$$f(x) \equiv 4x - 9, x \in \mathbb{R} \qquad g(x) \equiv x^2, x \in \mathbb{R}$$

a Evaluate $ff(3\frac{1}{4})$.

b Solve $gf(x) = 25$.

c Sketch the graph of $y = fg(x)$, showing the coordinates of any points of intersection with the coordinate axes.

C3 FUNCTIONS

Worksheet B continued

9 $f: x \rightarrow \tan x, x \in \mathbb{R}$ $g: x \rightarrow 4 + \ln x, x \in \mathbb{R}^+$ $h: x \rightarrow e^{2x-1}, x \in \mathbb{R}$

Evaluate

a $gf(\frac{\pi}{4})$ **b** $hg(e^{-2})$ **c** $gh(-1)$ **d** $ff(1)$
e $hf(0.2)$ **f** $fg(7)$ **g** $hh(\frac{1}{4})$ **h** $fg(e^e)$

10 $f: x \rightarrow 3e^x + 2, x \in \mathbb{R}$ $g: x \rightarrow 4x + 1, x \in \mathbb{R}$ $h: x \rightarrow \frac{1}{x+1}, x \in \mathbb{R}, x \neq -1$

Express the following composite functions in a similar form, stating the domain in each case.

a fg **b** gf **c** hf **d** gg
e hg **f** gh **g** hh **h** ggg

11 $f: x \rightarrow \sqrt{x+4}, x \in \mathbb{R}, x > -4$ $g: x \rightarrow e^{1+2x}, x \in \mathbb{R}$ $h: x \rightarrow \frac{x+1}{3}, x \in \mathbb{R}$

Solve

a $fh(x) = 3$ **b** $fg(x) = 7$ **c** $gh(x) = 11$ **d** $hh(x) = \frac{2}{3}$
e $hg(x) = 1.2$ **f** $hf(x) = \frac{1}{2}$ **g** $ff(x) = 3$ **h** $ghh(x) = \frac{1}{2}$

12 $f(x) \equiv x^3, x \in \mathbb{R}$ $g(x) \equiv x + 2, x \in \mathbb{R}$

Find the composition of the functions f and g that corresponds to the function h , where

a $h(x) \equiv (x+2)^3, x \in \mathbb{R}$ **b** $h(x) \equiv x^3 + 2, x \in \mathbb{R}$ **c** $h(x) \equiv x + 4, x \in \mathbb{R}$
d $h(x) \equiv x^9, x \in \mathbb{R}$ **e** $h(x) \equiv x^9 + 2, x \in \mathbb{R}$ **f** $h(x) \equiv (x+2)^3 + 2, x \in \mathbb{R}$

13 $f(x) \equiv x - 4, x \in \mathbb{R}$ $g(x) \equiv 3x^2, x \in \mathbb{R}$ $h(x) \equiv \frac{1}{x}, x \in \mathbb{R}, x \neq 0$

Find the composition of the functions f, g and h that corresponds to the function j , where

a $j(x) \equiv 3x^2 - 4, x \in \mathbb{R}$ **b** $j(x) \equiv \frac{1}{x-4}, x \in \mathbb{R}, x \neq 4$
c $j(x) \equiv \frac{3}{x^2}, x \in \mathbb{R}, x \neq 0$ **d** $j(x) \equiv 27x^4, x \in \mathbb{R}$
e $j(x) \equiv \frac{1}{3x^2} - 4, x \in \mathbb{R}, x \neq 0$ **f** $j(x) \equiv \frac{1}{3x^2 - 4}, x \in \mathbb{R}, x \neq \pm \frac{2}{\sqrt{3}}$

14 The functions f and g are defined by

$$f: x \rightarrow 5^x - 7, x \in \mathbb{R} \qquad g: x \rightarrow 2x + 3, x \in \mathbb{R}$$

- a** Find and simplify an expression for gf , stating its domain.
b Solve the equation $gf(x) = 10$.

15 The functions f and g are defined by

$$f: x \rightarrow 2(x+1), x \in \mathbb{R} \qquad g: x \rightarrow x^2 - 9, x \in \mathbb{R}$$

- a** Express gf in terms of x and state its domain and range.
b Sketch the graph of $y = gf(x)$, showing the coordinates of any points of intersection with the coordinate axes.

The equation $gf(x) - 2f(x) = a$, where a is a constant, has no real roots.

- c** Show that $a < -10$.