

C3 FUNCTIONS

Answers - Worksheet B

- 1** **a** $= g(5) = -3$ **b** $= g(6) = -4$ **c** $= f(5) = 17$ **d** $= h(9) = 86$
e $= g(-3) = 5$ **f** $= f(-1) = -7$ **g** $= h(-6) = 41$ **h** $= f\left(\frac{29}{4}\right) = 26$
- 2** **a** $= f(\ln 20) = 17.0$ **b** $= g(\ln 3) = 0.455$ **c** $= f(\cos 5) = 3.42$ **d** $= g(\cos(-4)) = 0.794$
e $= g\left(\frac{43}{4}\right) = -0.243$ **f** $= h(\cos 6.7) = -0.0895$ **g** $= h(\ln 50) = 1.36$ **h** $= h(0.5) = -0.693$
- 3** **a** $= f(1 - 3x)$
 $= 2(1 - 3x) + 1$
fg : $x \rightarrow 3 - 6x,$
 $x \in \mathbb{R}$
b $= f(2x + 1)$
 $= 2(2x + 1) + 1$
ff : $x \rightarrow 4x + 3,$
 $x \in \mathbb{R}$
c $= f(x^2 + 4)$
 $= 2(x^2 + 4) + 1$
fh : $x \rightarrow 2x^2 + 9,$
 $x \in \mathbb{R}$
d $= h(2x + 1)$
 $= (2x + 1)^2 + 4$
hf : $x \rightarrow 4x^2 + 4x + 5,$
 $x \in \mathbb{R}$
e $= g(x^2 + 4)$
 $= 1 - 3(x^2 + 4)$
gh : $x \rightarrow -3x^2 - 11,$
 $x \in \mathbb{R}$
f $= g(1 - 3x)$
 $= 1 - 3(1 - 3x)$
gg : $x \rightarrow 9x - 2,$
 $x \in \mathbb{R}$
g $= h(1 - 3x)$
 $= (1 - 3x)^2 + 4$
hg : $x \rightarrow 9x^2 - 6x + 5,$
 $x \in \mathbb{R}$
h $= g(2x + 1)$
 $= 1 - 3(2x + 1)$
gf : $x \rightarrow -6x - 2,$
 $x \in \mathbb{R}$
- 4** **a** $= g(4 - x)$
 $= e^{4-x}$
gf : $x \rightarrow e^{4-x},$
 $x \in \mathbb{R}$
b $= h(e^x)$
 $= 2(e^x)^2 + 7$
hg : $x \rightarrow 2e^{2x} + 7,$
 $x \in \mathbb{R}$
c $= f(2x^2 + 7)$
 $= 4 - (2x^2 + 7)$
fh : $x \rightarrow -2x^2 - 3,$
 $x \in \mathbb{R}$
d $= g(e^x)$
 $= e^{e^x}$
gg : $x \rightarrow e^{e^x},$
 $x \in \mathbb{R}$
e $= g(2x^2 + 7)$
 $= e^{2x^2+7}$
gh : $x \rightarrow e^{2x^2+7},$
 $x \in \mathbb{R}$
f $= f(4 - x)$
 $= 4 - (4 - x)$
ff : $x \rightarrow x,$
 $x \in \mathbb{R}$
g $= f(e^x)$
 $= 4 - e^x$
fg : $x \rightarrow 4 - e^x,$
 $x \in \mathbb{R}$
h $= h(4 - x)$
 $= 2(4 - x)^2 + 7$
hf : $x \rightarrow 2x^2 - 16x + 39,$
 $x \in \mathbb{R}$
- 5** **a** **ff**(x) $= f(5x - 3)$
 $= 5(5x - 3) - 3$
 $= 25x - 18$
 $25x - 18 = -8$
 $x = \frac{2}{5}$
b **hf**(x) $= h(5x - 3)$
 $= \frac{1}{5x - 3 - 2}$
 $= \frac{1}{5x - 5}$
 $\frac{1}{5x - 5} = 2$
 $1 = 10x - 10$
 $x = \frac{11}{10}$
c **gf**(x) $= g(5x - 3)$
 $= 3(5x - 3)^2 + 1$
 $= 75x^2 - 90x + 28$
 $75x^2 - 90x + 28 = 28$
 $15x(5x - 6) = 0$
 $x = 0, \frac{6}{5}$
d **hg**(x) $= h(3x^2 + 1)$
 $= \frac{1}{3x^2 + 1 - 2}$
 $= \frac{1}{3x^2 - 1}$
 $\frac{1}{3x^2 - 1} = \frac{1}{2}$
 $3x^2 - 1 = 2$
 $x^2 = 1$
 $x = \pm 1$
- e** **fh**(x) $= f\left(\frac{1}{x-2}\right)$
 $= \frac{5}{x-2} - 3$
 $\frac{5}{x-2} - 3 = 7$
 $\frac{5}{x-2} = 10$
 $5 = 10x - 20$
 $x = \frac{5}{2}$
f **fg**(x) $= f(3x^2 + 1)$
 $= 5(3x^2 + 1) - 3$
 $= 15x^2 + 2$
 $15x^2 + 2 = 32$
 $x^2 = 2$
 $x = \pm\sqrt{2}$
g **gh**(x) $= g\left(\frac{1}{x-2}\right)$
 $= \frac{3}{(x-2)^2} + 1$
 $\frac{3}{(x-2)^2} + 1 = 4$
 $(x-2)^2 = 1$
 $x - 2 = \pm 1$
 $x = 1, 3$
h **hh**(x) $= h\left(\frac{1}{x-2}\right)$
 $= \frac{1}{\frac{1}{x-2} - 2} = \frac{x-2}{1-2(x-2)}$
 $= \frac{x-2}{5-2x}$
 $\frac{x-2}{5-2x} = -2$
 $x - 2 = -10 + 4x$
 $x = \frac{8}{3}$

6	a	$gh(x) = g(e^x)$	b	$fg(x) = f(3 + 2x)$	c	$hg(x) = h(3 + 2x)$	d	$gf(x) = g(\ln x)$
		$= 3 + 2e^x$		$= \ln(3 + 2x)$		$= e^{3+2x}$		$= 3 + 2 \ln x$
		$3 + 2e^x = 9$		$\ln(3 + 2x) = 3.6$		$e^{3+2x} = 4$		$3 + 2 \ln x = 10.4$
		$e^x = 3$		$3 + 2x = e^{3.6}$		$3 + 2x = \ln 4$		$\ln x = 3.7$
		$x = \ln 3$		$x = \frac{1}{2}(e^{3.6} - 3)$		$x = \frac{1}{2}(\ln 4 - 3)$		$x = e^{3.7}$
		$x = 1.10$		$x = 16.80$		$x = -0.81$		$x = 40.45$

7 a $g(x) > 0$

b $fg(x) = f(e^x) = \frac{e^x + 1}{5}$

$$\frac{e^x + 1}{5} = 17$$

$$e^x + 1 = 85$$

$$e^x = 84$$

$$x = \ln 84 = 4.43 \text{ (3sf)}$$

8 a $= f(4) = 7$

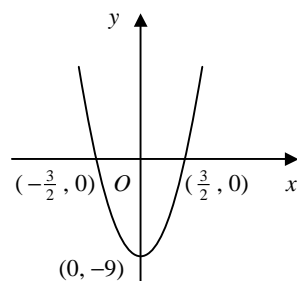
b $gf(x) = g(4x - 9) = (4x - 9)^2$

$$\therefore (4x - 9)^2 = 25$$

$$4x - 9 = \pm 5$$

$$x = \frac{1}{4}(9 \pm 5) = 1, \frac{7}{2}$$

c $fg(x) = f(x^2) = 4x^2 - 9$



9 a $= g(1) = 4$

b $= h(2) = e^3$

c $= g(e^{-3}) = 1$

d $= f(\tan 1) = 74.7 \text{ (3sf)}$

e $= h(\tan 0.2)$
 $= 0.552 \text{ (3sf)}$

f $= f(4 + \ln 7)$
 $= -0.351 \text{ (3sf)}$

g $= h(e^{-\frac{1}{2}})$
 $= 1.24 \text{ (3sf)}$

h $= f(4 + e)$
 $= 0.465 \text{ (3sf)}$

10 a $= f(4x + 1)$
 $= 3e^{4x+1} + 2$

b $= g(3e^x + 2)$
 $= 4(3e^x + 2) + 1$

c $= h(3e^x + 2)$
 $= \frac{1}{3e^x + 2 + 1}$

d $= g(4x + 1)$
 $= 4(4x + 1) + 1$

$fg : x \rightarrow 3e^{4x+1} + 2,$
 $x \in \mathbb{R}$

$gf : x \rightarrow 12e^x + 9,$
 $x \in \mathbb{R}$

$hf : x \rightarrow \frac{1}{3e^x + 3},$
 $x \in \mathbb{R}$

$gg : x \rightarrow 16x + 5,$
 $x \in \mathbb{R}$

e $= h(4x + 1)$
 $= \frac{1}{4x+1+1}$

f $= g\left(\frac{1}{x+1}\right)$
 $= \frac{4}{x+1} + 1$

g $= h\left(\frac{1}{x+1}\right)$
 $= \frac{1}{\frac{1}{x+1} + 1}$

h $= g(16x + 5)$
 $= 4(16x + 5) + 1$

$hg : x \rightarrow \frac{1}{4x+2},$
 $x \in \mathbb{R}, x \neq -\frac{1}{2}$

$= \frac{4+x+1}{x+1}$
 $gh : x \rightarrow \frac{x+5}{x+1},$
 $x \in \mathbb{R}, x \neq -1$

$= \frac{x+1}{1+x+1}$
 $hh : x \rightarrow \frac{x+1}{x+2},$
 $x \in \mathbb{R}, x \neq -1, -2$

$ggg : x \rightarrow 64x + 21,$
 $x \in \mathbb{R}$

11 a $fh(x) = f\left(\frac{x+1}{3}\right)$ **b** $fg(x) = f(e^{1+2x})$ **c** $gh(x) = g\left(\frac{x+1}{3}\right)$ **d** $hh(x) = h\left(\frac{x+1}{3}\right)$

$$= \sqrt{\frac{x+1}{3} + 4}$$

$$= \sqrt{\frac{x+13}{3}}$$

$$\sqrt{\frac{x+13}{3}} = 3$$

$$\frac{x+13}{3} = 9$$

$$x + 13 = 27$$

$$x = 14$$

$$= \sqrt{e^{1+2x} + 4}$$

$$\sqrt{e^{1+2x} + 4} = 7$$

$$e^{1+2x} + 4 = 49$$

$$e^{1+2x} = 45$$

$$1 + 2x = \ln 45$$

$$x = \frac{1}{2}(\ln 45 - 1)$$

$$x = 1.40 \text{ (3sf)}$$

$$= e^{1+\frac{2(x+1)}{3}}$$

$$= e^{\frac{2x+5}{3}}$$

$$e^{\frac{2x+5}{3}} = 11$$

$$\frac{2x+5}{3} = \ln 11$$

$$2x + 5 = 3 \ln 11$$

$$x = \frac{1}{2}(3 \ln 11 - 5)$$

$$x = 1.10 \text{ (3sf)}$$

$$= \frac{\frac{x+1}{3} + 1}{3}$$

$$= \frac{x+1+3}{9}$$

$$= \frac{x+4}{9}$$

$$\frac{x+4}{9} = \frac{2}{3}$$

$$3x + 12 = 18$$

$$x = 2$$

e $hg(x) = h(e^{1+2x})$ **f** $hf(x) = h(\sqrt{x+4})$ **g** $ff(x) = f(\sqrt{x+4})$ **h** $ghh(x) = g\left(\frac{x+4}{9}\right)$

$$= \frac{e^{1+2x} + 1}{3}$$

$$\frac{e^{1+2x} + 1}{3} = 1.2$$

$$e^{1+2x} = 2.6$$

$$1 + 2x = \ln 2.6$$

$$x = \frac{1}{2}(\ln 2.6 - 1)$$

$$x = -0.0222 \text{ (3sf)}$$

$$= \frac{\sqrt{x+4} + 1}{3}$$

$$\frac{\sqrt{x+4} + 1}{3} = \frac{1}{2}$$

$$\sqrt{x+4} = \frac{1}{2}$$

$$x + 4 = \frac{1}{4}$$

$$x = -3\frac{3}{4}$$

$$= \sqrt{\sqrt{x+4} + 4}$$

$$\sqrt{\sqrt{x+4} + 4} = 3$$

$$\sqrt{x+4} = 5$$

$$x + 4 = 25$$

$$x = 21$$

$$= e^{1+\frac{2(x+4)}{9}}$$

$$= e^{\frac{2x+17}{9}}$$

$$e^{\frac{2x+17}{9}} = \frac{1}{2}$$

$$\frac{2x+17}{9} = \ln \frac{1}{2}$$

$$x = \frac{1}{2}(9 \ln \frac{1}{2} - 17)$$

$$x = -11.6 \text{ (3sf)}$$

12 a $h(x) = fg(x)$

b $h(x) = gf(x)$

c $h(x) = gg(x)$

d $h(x) = ff(x)$

e $h(x) = gff(x)$

f $h(x) = gfg(x)$

13 a $j(x) = fg(x)$

b $j(x) = hf(x)$

c $j(x) = gh(x)$

d $j(x) = gg(x)$

e $j(x) = fhg(x)$

f $j(x) = hfg(x)$

14 a $gf(x) = g(5^x - 7)$
 $= 2(5^x - 7) + 3$
 $= 2(5^x) - 11$
 $gf : x \rightarrow 2(5^x) - 11, x \in \mathbb{R}$

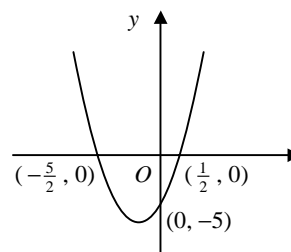
b $2(5^x) - 11 = 10$

$$5^x = \frac{21}{2}$$

$$x = \frac{\ln \frac{21}{2}}{\ln 5} = 1.46 \text{ (3sf)}$$

15 a $gf(x) = g[2(x+1)] = [2(x+1)]^2 - 9$
 $gf : x \rightarrow 4x^2 + 8x - 5, x \in \mathbb{R}$
range: $gf(x) \geq -9$

b $gf(x) = (2x+5)(2x-1)$



c $gf(x) - 2f(x) = a$

$$4x^2 + 8x - 5 - 2[2(x+1)] = a$$

$$4x^2 + 4x - (a+9) = 0$$

no real roots $\therefore b^2 - 4ac < 0$

$$16 + 16(a+9) < 0$$

$$1 + a + 9 < 0$$

$$a < -10$$