

C3

EXPONENTIALS AND LOGARITHMS

Worksheet A

- 1 Find, to 3 significant figures, the value of
 a e^3 b e^{-2} c $5e$ d $\ln 0.55$ e $\frac{3}{7}\ln 100$ f $\log_{10} e$
- 2 Without using a calculator, find the value of
 a $e^{\ln 4}$ b $e^{\frac{1}{3}\ln 9}$ c $2e^{-\ln 6}$ d $\ln e^7$ e $\ln \frac{1}{e}$ f $5 \ln e^{-0.1}$
- 3 Find the value of x in each case.
 a $e^{\ln x} = 4$ b $\ln e^x = 17$ c $e^{2\ln x} = 25$ d $e^{-\ln x} = \frac{1}{3}$
- 4 Solve each equation, giving your answers in terms of e .
 a $\ln x = 15$ b $\frac{1}{2}\ln t - 3 = 0$ c $\ln(x - 4) = 7$
 d $17 - \ln 5y = 9$ e $\ln(\frac{1}{2}x + 3) = 2.5$ f $\ln(4 - 3x) - 11 = 0$
- 5 Solve each equation, giving your answers in terms of natural logarithms.
 a $e^x = 0.7$ b $9 - 2e^y = 5$ c $e^{5x} - 3 = 0$
 d $e^{4t+1} = 12$ e $\frac{1}{2}e^{2x-3} - 7 = 0$ f $2e^{4-5x} + 9 = 16$
- 6 Solve each equation, giving your answers to 2 decimal places.
 a $\frac{1}{3}e^x = 4$ b $\ln(15x - 7) = 4$ c $4e^{\frac{1}{2}y+3} = 11$
 d $\frac{3}{7}\ln(5 - 2x) - 1 = 0$ e $\ln(10 - 3y) - e = 0$ f $\ln x^2 + \ln x^3 = 19$
 g $e^{2x} = 3e^{-\frac{1}{4}x}$ h $e^{5t} = 4e^{2t+1}$ i $\ln(2x - 5) - \ln x = \frac{1}{4}$
- 7 Find, in exact form, the solutions to the equation
 $2e^{2x} + 12 = 11e^x$.
- 8 a Simplify

$$\frac{3x^2 - 10x + 8}{x^2 - 5x + 6}$$

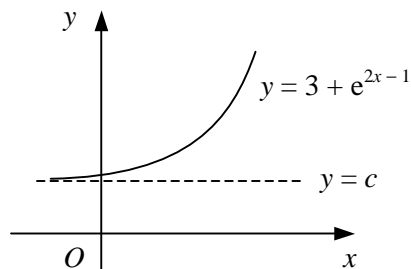
 b Hence, solve the equation
 $\ln(3x^2 - 10x + 8) - \ln(x^2 - 5x + 6) = \ln 2x$.
- 9 Solve the following simultaneous equations, giving your answers to 2 decimal places.
 $e^{5y} - x = 0$
 $\ln x^4 = 7 - y$
- 10 Sketch each pair of curves on the same diagram, showing the coordinates of any points of intersection with the coordinate axes.
 a $y = e^x$
 $y = e^{-2x}$ b $y = 2e^x$
 $y = e^{x-1}$ c $y = 2 + e^x$
 $y = e^{2x+1}$
 d $y = e^x$
 $y = \ln x$ e $y = -\ln x$
 $y = 2 + \ln x$ f $y = \ln(x - 2)$
 $y = \ln 3x$

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Worksheet A continued

- 11 a Sketch on the same diagram the curves $y = \ln(x + 1)$ and $y = 1 + \ln x$.
 b Show that the x -coordinate of the point where the two curves intersect is $\frac{1}{e-1}$.

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The diagram shows the curve with the equation $y = 3 + e^{2x-1}$ and the asymptote of the curve which has the equation $y = c$.

- a State the value of the constant c .
 b Find the exact coordinates of the point where the curve crosses the y -axis.
 c Find the x -coordinate of the point on the curve where $y = 7$, giving your answer in the form $a + \ln b$, where a is rational and b is an integer.
- 13 A quantity N is decreasing such that at time t
- $$N = 50e^{-0.2t}.$$
- a Find the value of N when $t = 10$.
 b Find the value of t when $N = 3$.
- 14 A radioactive substance is decaying such that its mass, m grams, at a time t years after initial observation is given by
- $$m = 240e^{kt},$$
- where k is a constant.
 Given that when $t = 180$, $m = 160$, find
- a the value of k ,
 b the time it takes for the mass of the substance to be halved.
- 15 A quantity N is increasing such that at time t
- $$N = 20e^{0.04t}.$$
- a Find the value of N when $t = 15$.
 b Find, in terms of the constant k , expressions for the value of t when
- $N = k$,
 - $N = 2k$.
- c Hence, show that the time it takes for the value of N to double is constant.
- 16 A quantity N is decreasing such that at time t
- $$N = N_0e^{kt}.$$
- Given that at time $t = 10$, $N = 300$ and that at time $t = 20$, $N = 225$, find
- a the values of the constants N_0 and k ,
 b the value of t when $N = 150$.