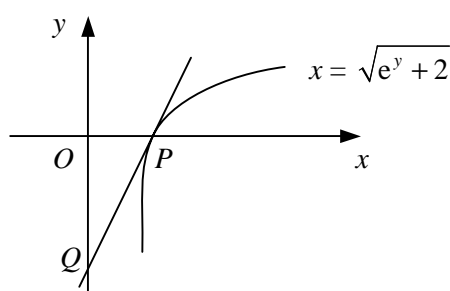


C3 DIFFERENTIATION

Worksheet K

- 1 The curve C has equation $y = \frac{1}{4x} - \ln x$.
- a Find the gradient of C at the point $(1, \frac{1}{4})$. (3)
- b Find an equation for the normal to C at the point $(1, \frac{1}{4})$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. (3)
- 2 A curve has the equation $y = xe^{-2x}$.
- a Find and simplify expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. (4)
- b Find the exact coordinates of the turning point of the curve and determine its nature. (4)

3



The diagram shows the curve $x = \sqrt{e^y + 2}$ which crosses the x -axis at the point P .

- a Find the coordinates of P . (1)
- b Find $\frac{dx}{dy}$ in terms of y . (2)
- The tangent to the curve at P crosses the y -axis at the point Q .
- c Show that the area of triangle OPQ , where O is the origin, is $3\sqrt{3}$. (5)
- 4 A rock contains a radioactive substance which is decaying.
- The mass of the rock, m grams, at time t years after initial observation is given by
- $$m = 600 + 80e^{-0.004t}.$$
- a Find the percentage reduction in the mass of the rock over the first 100 years. (3)
- b Find the value of t when $m = 640$. (2)
- c Find the rate at which the mass of the rock will be decreasing when $t = 150$. (3)

5 Differentiate with respect to x

- a $\sqrt{\sin x + \cos x}$, (3)
- b $\ln\left(\frac{x-1}{2x+1}\right)$. (3)

6 A curve has the equation $y = (2x - 3)^5$.

- a Find an equation for the tangent to the curve at the point $P(1, -1)$. (4)
- Given that the tangent to the curve at the point Q is parallel to the tangent at P ,
- b find the coordinates of Q . (3)

C3 DIFFERENTIATION

Worksheet K continued

- 7 A curve has the equation $y = \frac{2}{x^2 - 5}$.
- a Find the coordinates of the stationary point of the curve. (4)
- b Show that the tangent to the curve at the point with x -coordinate 3 has the equation $3x + 4y - 11 = 0$. (3)
- 8 $f: x \rightarrow ae^x + a, x \in \mathbb{R}$.
- Given that a is a positive constant,
- a sketch the graph of $y = f(x)$, showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes. (2)
- b Find the inverse function f^{-1} in the form $f^{-1}: x \rightarrow \dots$ and state its domain. (4)
- c Find an equation for the tangent to the curve $y = f(x)$ at the point on the curve with x -coordinate 1. (4)
- 9 a Use the derivatives of $\sin x$ and $\cos x$ to prove that $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$. (4)
- b Show that the curve with equation $y = e^x \cot x$ has no turning points. (5)
- 10 A curve has the equation $y = (2 + \ln x)^3$.
- a Find $\frac{dy}{dx}$. (2)
- b Find, in exact form, the coordinates of the stationary point on the curve. (3)
- c Show that the tangent to the curve at the point with x -coordinate e passes through the origin. (3)
- 11 $f: x \rightarrow \ln(9 - x^2), -3 < x < 3$.
- a Find $f'(x)$. (2)
- b Find the coordinates of the stationary point of the curve $y = f(x)$. (2)
- c Show that the normal to the curve $y = f(x)$ at the point with x -coordinate 1 has equation $y = 4x - 4 + 3 \ln 2$. (4)
- 12 A botanist is studying the regeneration of an area of moorland following a fire. The total biomass in the area after t years is denoted by M tonnes and two models are proposed for the growth of M .
- Model A is given by $M = 900 - \frac{1500}{3t + 2}$.
- Model B is given by $M = 900 - \frac{1500}{2 + 5 \ln(t + 1)}$.
- For each model, find
- a the value of M when $t = 3$, (2)
- b the rate at which the biomass is increasing when $t = 3$. (6)