C3 DIFFERENTIATION Worksheet J

1 Find an equation for the tangent to the curve with equation

$$y = (3 - x)^{\frac{3}{2}}$$

at the point on the curve with x-coordinate -1. **(4)**

2 **a** Sketch the curve with equation $y = 3 - \ln 2x$. **(2)**

b Find the exact coordinates of the point where the curve crosses the x-axis. **(2)**

c Find an equation for the tangent to the curve at the point on the curve where $x = 5$. **(4)**

This tangent cuts the x-axis at A and the y-axis at B.

d Show that the area of triangle OAB, where O is the origin, is approximately 7.20 **(3)**

3 Differentiate with respect to x

a $(3x - 1)^4$, **(2)**

b $\frac{x^2}{\sin 2x}$. **(3)**

4 The area of the surface of a boulder covered by lichen, A cm^2, at time t years after initial observation, is modelled by the formula

$$A = 2e^{0.5t}.$$

Using this model,

a find the area of lichen on the boulder after three years, **(2)**

b find the rate at which the area of lichen is increasing per day after three years, **(4)**

c find, to the nearest year, how long it takes until the area of lichen is 65 cm^2. **(2)**

d Explain why the model cannot be valid for large values of t. **(1)**

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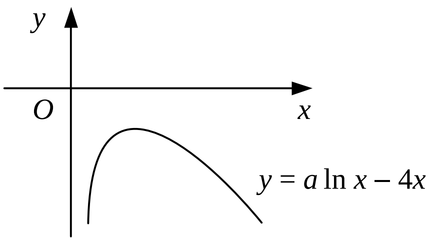

The diagram shows the curve with equation $y = a \ln x - 4x$, where a is a positive constant.

Find, in terms of a,

a the coordinates of the stationary point on the curve, **(4)**

b an equation for the tangent to the curve at the point where $x = 1$. **(3)**

Given that this tangent meets the x-axis at the point (3, 0),

c show that $a = 6$. **(2)**

6 Given that $y = e^{2x} \sin x$,

a find $\frac{dy}{dx}$, **(2)**

b show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$. **(3)**

7 A curve has the equation $x = \tan^2 y$.

a Show that $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}(x+1)}$. **(5)**

b Find an equation for the normal to the curve at the point where $y = \frac{\pi}{4}$. **(3)**

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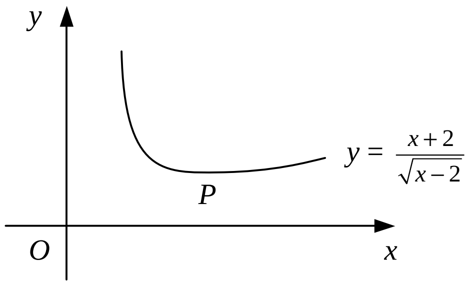

The diagram shows the curve $y = \dfrac{x+2}{\sqrt{x-2}}$, $x > 2$, which has a minimum point at P.

a Find and simplify an expression for $\dfrac{dy}{dx}$. **(3)**

b Find the coordinates of P. **(2)**

The point Q on the curve has x-coordinate 3.

c Show that the normal to the curve at Q has equation

$$2x - 3y + 9 = 0.$$ **(3)**

9 A curve has the equation $y = e^x(x - 1)^2$.

a Find $\dfrac{dy}{dx}$. **(3)**

b Show that $\dfrac{d^2y}{dx^2} = e^x(x^2 + 2x - 1)$. **(2)**

c Find the exact coordinates of the turning points of the curve and determine their nature. **(4)**

d Show that the tangent to the curve at the point where $x = 2$ has the equation

$$y = e^2(3x - 5).$$ **(3)**

10 The curve with equation $y = \frac{1}{2}x^2 - 3\ln x$, $x > 0$, has a stationary point at A.

a Find the exact x-coordinate of A. **(3)**

b Determine the nature of the stationary point. **(2)**

c Show that the y-coordinate of A is $\frac{3}{2}(1 - \ln 3)$. **(2)**

d Find an equation for the tangent to the curve at the point where $x = 1$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

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$$f(x) = \frac{6x}{(x-1)(x+2)} - \frac{2}{x-1}.$$

a Show that $f(x) = \dfrac{4}{x+2}$. **(5)**

b Find an equation for the tangent to the curve $y = f(x)$ at the point with x-coordinate 2, giving your answer in the form $ax + by = c$, where a, b and c are integers. **(4)**