DIFFERENTIATION

1 A curve has the equation $x = \sqrt{y}$.

C3

- **a** Write down $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.
- **b** Express the equation of the curve in the form y = f(x).
- **c** Write down $\frac{dy}{dx}$ in terms of *x*.

d Hence verify that for this curve, $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.

2 Verify the relationship $\frac{dy}{dx} \times \frac{dx}{dy} = 1$ when

- **a** $y = e^{2x-1}$, **b** $y = x^3 + 2$.
- c $x = \sqrt{\ln y}$.

3 Find expressions for $\frac{dy}{dx}$ in terms of y in each case.

a	$x = y^2 + 3$	b	$x = (y - 1)^3$	c	$x = \tan y$
d	$x = \ln (3y + 2)$	e	$x = \sin^2 y$	f	$x = \frac{y-2}{e^y}$

4 The curve C has the equation
$$x = y^3 - 4y^2$$
.
a Find $\frac{dx}{dy}$ in terms of y.

- **b** Find an equation for the tangent to *C* at the point on the curve with *y*-coordinate 3.
- 5 Given that $y = \ln (ax + b)$, where *a* and *b* are constants,
 - **a** express x as a function of y,
 - **b** find $\frac{dx}{dy}$ in terms of y.

c Hence, prove that
$$\frac{d}{dx} [\ln (ax + b)] = \frac{a}{ax+b}$$

- 6 A curve has the equation $y = 3^x$.
 - **a** Express the equation of the curve in the form x = f(y).
 - **b** Find $\frac{dx}{dy}$ in terms of y. **c** Hence, find $\frac{dy}{dx}$ in terms of x.
 - **d** Find an equation for the tangent to the curve at the point (2, 9).