

### C3 DIFFERENTIATION

### Answers - Worksheet H

- 1**
- a**  $= -\sin x$
- c**  $= -3 \sin 3x$
- e**  $= \cos(x + 1)$
- g**  $= -4 \cos\left(\frac{\pi}{3} - x\right)$
- i**  $= 2 \sin x \cos x$
- k**  $= 2 \cos(x - 1) \times [-\sin(x - 1)]$   
 $= -2 \cos(x - 1) \sin(x - 1)$
- b**  $= 5 \cos x$
- d**  $= \frac{1}{4} \cos \frac{1}{4}x$
- f**  $= -3 \sin(3x - 2)$
- h**  $= -\frac{1}{2} \sin\left(\frac{1}{2}x + \frac{\pi}{6}\right)$
- j**  $= 6 \cos^2 x \times (-\sin x)$   
 $= -6 \cos^2 x \sin x$
- l**  $= 4 \sin^3 2x \times 2 \cos 2x$   
 $= 8 \sin^3 2x \cos 2x$
- 2**
- a**  $= \frac{d}{dx} \left( \frac{\sin x}{\cos x} \right)$   
 $= \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$   
 $= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$   
 $= \frac{1}{\cos^2 x}$   
 $= \sec^2 x$
- b**  $= \frac{d}{dx} [(\cos x)^{-1}]$   
 $= -(\cos x)^{-2} \times (-\sin x)$   
 $= \frac{\sin x}{\cos^2 x}$   
 $= \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$   
 $= \sec x \tan x$
- c**  $= \frac{d}{dx} [(\sin x)^{-1}]$   
 $= -(\sin x)^{-2} \times \cos x$   
 $= -\frac{\cos x}{\sin^2 x}$   
 $= -\frac{1}{\sin x} \times \frac{\cos x}{\sin x}$   
 $= -\operatorname{cosec} x \cot x$
- d**  $= \frac{d}{dx} \left( \frac{\cos x}{\sin x} \right)$   
 $= \frac{-\sin x \times \sin x - \cos x \times \cos x}{\sin^2 x}$   
 $= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}$   
 $= -\frac{1}{\sin^2 x}$   
 $= -\operatorname{cosec}^2 x$
- 3**
- a**  $= -2 \operatorname{cosec}^2 2t$
- c**  $= 4 \sec^2(4t - 3)$
- e**  $= 2 \tan t \times \sec^2 t$   
 $= 2 \tan t \sec^2 t$
- g**  $= 3 \cot^2 t \times (-\operatorname{cosec}^2 t)$   
 $= -3 \cot^2 t \operatorname{cosec}^2 t$
- i**  $= -2 \operatorname{cosec}^2(2t - 3)$
- k**  $= -2 \sec^2(\pi - 4t)$
- b**  $= \sec(t + 2) \tan(t + 2)$
- d**  $= -3 \operatorname{cosec} 3t \cot 3t$
- f**  $= -3 \operatorname{cosec}\left(t + \frac{\pi}{6}\right) \cot\left(t + \frac{\pi}{6}\right)$
- h**  $= 2 \sec \frac{1}{2}t \tan \frac{1}{2}t$
- j**  $= 2 \sec 2t \times 2 \sec 2t \tan 2t$   
 $= 4 \sec^2 2t \tan 2t$
- l**  $= 2 \operatorname{cosec}(3t + 1) \times -3 \operatorname{cosec}(3t + 1) \cot(3t + 1)$   
 $= -6 \operatorname{cosec}^2(3t + 1) \cot(3t + 1)$

- 4**
- a**  $= \frac{1}{\sin x} \times \cos x$   
 $= \cot x$
- b**  $= 6e^{\tan x} \times \sec^2 x$   
 $= 6e^{\tan x} \sec^2 x$
- c**  $= \frac{1}{2}(\cos 2x)^{-\frac{1}{2}} \times (-2 \sin 2x)$   
 $= -\frac{\sin 2x}{\sqrt{\cos 2x}}$
- d**  $= e^{\sin 3x} \times 3 \cos 3x$   
 $= 3e^{\sin 3x} \cos 3x$
- e**  $= -2 \operatorname{cosec}^2 x^2 \times 2x$   
 $= -4x \operatorname{cosec}^2 x^2$
- f**  $= \frac{1}{2}(\sec x)^{-\frac{1}{2}} \times \sec x \tan x$   
 $= \frac{1}{2}\sqrt{\sec x} \tan x$
- g**  $= 3e^{-\operatorname{cosec} 2x} \times 2 \operatorname{cosec} 2x \cot 2x$   
 $= 6 \operatorname{cosec} 2x \cot 2x e^{-\operatorname{cosec} 2x}$
- h**  $= \frac{1}{\tan 4x} \times 4 \sec^2 4x$   
 $= \frac{\cos 4x}{\sin 4x} \times \frac{4}{\cos^2 4x}$   
 $= 4 \sec 4x \operatorname{cosec} 4x$
- 5**
- a**  $\frac{dy}{dx} = 1 + 2 \cos x$   
 SP:  $1 + 2 \cos x = 0$   
 $\cos x = -\frac{1}{2}$   
 $x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$   
 $x = \frac{2\pi}{3}, \frac{4\pi}{3}$   
 $\therefore (\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3}),$   
 $(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3})$
- b**  $\frac{dy}{dx} = 2 \sec x \tan x - \sec^2 x$   
 $= \sec x(2 \tan x - \sec x)$   
 SP:  $\sec x(2 \tan x - \sec x) = 0$   
 $2 \tan x - \sec x = 0$   
 $\frac{2 \sin x}{\cos x} = \frac{1}{\cos x}$   
 $\sin x = \frac{1}{2}$   
 $x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$   
 $x = \frac{\pi}{6}, \frac{5\pi}{6}$   
 $\therefore (\frac{\pi}{6}, \sqrt{3}), (\frac{5\pi}{6}, -\sqrt{3})$
- c**  $\frac{dy}{dx} = \cos x - 2 \sin 2x$   
 SP:  $\cos x - 2 \sin 2x = 0$   
 $\cos x - 4 \sin x \cos x = 0$   
 $\cos x(1 - 4 \sin x) = 0$   
 $\cos x = 0$  or  $\sin x = \frac{1}{4}$   
 $x = \frac{\pi}{2}, \frac{3\pi}{2}$  or  $0.253, \pi - 0.25268$   
 $x = 0.253$  (3sf),  $\frac{\pi}{2}, 2.89$  (3sf),  $\frac{3\pi}{2}$   
 $\therefore (0.253, \frac{9}{8}), (\frac{\pi}{2}, 0),$   
 $(2.89, \frac{9}{8}), (\frac{3\pi}{2}, -2)$
- 6**
- a**  $x = 0 \therefore y = 1$   
 $\frac{dy}{dx} = 2 \cos 2x$   
 grad = 2  
 $\therefore y - 1 = 2(x - 0)$   
 $[y = 2x + 1]$
- b**  $x = \frac{\pi}{3} \therefore y = \frac{1}{2}$   
 $\frac{dy}{dx} = -\sin x$   
 grad =  $-\frac{\sqrt{3}}{2}$   
 $\therefore y - \frac{1}{2} = -\frac{\sqrt{3}}{2}(x - \frac{\pi}{3})$   
 $[3\sqrt{3}x + 6y - 3 - \sqrt{3}\pi = 0]$
- c**  $x = \frac{\pi}{4} \therefore y = -1$   
 $\frac{dy}{dx} = 3 \sec^2 3x$   
 grad = 6  
 $\therefore y + 1 = 6(x - \frac{\pi}{4})$   
 $[12x - 2y - 2 - 3\pi = 0]$
- d**  $x = \frac{\pi}{6} \therefore y = 1$   
 $\frac{dy}{dx} = -\operatorname{cosec} x \cot x - 2 \cos x$   
 grad =  $-3\sqrt{3}$   
 $\therefore y - 1 = -3\sqrt{3}(x - \frac{\pi}{6})$   
 $[6\sqrt{3}x + 2y - 2 - \sqrt{3}\pi = 0]$

$$7 \quad \mathbf{a} = 1 \times \sin x + x \times \cos x \\ = \sin x + x \cos x$$

$$\mathbf{c} = e^x \times \cos x + e^x \times (-\sin x) \\ = e^x(\cos x - \sin x)$$

$$\mathbf{e} = 2x \times \operatorname{cosec} x + x^2 \times (-\operatorname{cosec} x \cot x) \\ = x \operatorname{cosec} x(2 - x \cot x)$$

$$\mathbf{g} = \frac{1 \times \tan x - x \times \sec^2 x}{\tan^2 x} \\ = \cot x - x \operatorname{cosec}^2 x$$

$$\mathbf{i} = 2 \cos x(-\sin x) \times \cot x + \cos^2 x \times (-\operatorname{cosec}^2 x) \quad \mathbf{j} = \frac{2 \sec 2x \tan 2x \times x^2 - \sec 2x \times 2x}{x^4} \\ = -2 \cos^2 x - \cot^2 x \quad = \frac{2 \sec 2x(x \tan 2x - 1)}{x^3}$$

$$\mathbf{k} = 1 \times \tan^2 4x + x \times 2 \tan 4x \times 4 \sec^2 4x \\ = \tan 4x(\tan 4x + 8x \sec^2 4x)$$

$$\mathbf{b} = \frac{-2 \sin 2x \times x - \cos 2x \times 1}{x^2} \\ = -\frac{2x \sin 2x + \cos 2x}{x^2}$$

$$\mathbf{d} = \cos x \times \cos x + \sin x \times (-\sin x) \\ = \cos^2 x - \sin^2 x = \cos 2x$$

$$\mathbf{f} = \sec x \tan x \times \tan x + \sec x \times \sec^2 x \\ = \sec x(\tan^2 x + \sec^2 x)$$

$$\mathbf{h} = \frac{2 \cos 2x \times e^{3x} - \sin 2x \times 3e^{3x}}{(e^{3x})^2} \\ = \frac{2 \cos 2x - 3 \sin 2x}{e^{3x}}$$

$$\mathbf{l} = \frac{\cos x \times \cos 2x - \sin x \times (-2 \sin 2x)}{\cos^2 2x} \\ = \frac{\cos x(1 - 2 \sin^2 x) + 4 \sin^2 x \cos x}{\cos^2 2x} = \frac{\cos x(1 + 2 \sin^2 x)}{\cos^2 2x}$$

$$8 \quad \mathbf{a} \quad f'(x) = 3 \cos 3x \times \cos 5x + \sin 3x \times (-5 \sin 5x) \quad \mathbf{b} \quad f'(x) = 2 \sec^2 2x \times \sin x + \tan 2x \times \cos x \\ = 3 \cos 3x \cos 5x - 5 \sin 3x \sin 5x \\ = 2 \sec^2 2x \sin x + \tan 2x \cos x$$

$$f'\left(\frac{\pi}{4}\right) = 3\left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) - 5 \times \frac{1}{\sqrt{2}} \left(-\frac{1}{\sqrt{2}}\right) \\ = \frac{3}{2} + \frac{5}{2} = 4$$

$$f'\left(\frac{\pi}{3}\right) = 2 \times 4 \times \frac{\sqrt{3}}{2} + (-\sqrt{3}) \times \frac{1}{2} \\ = 4\sqrt{3} - \frac{1}{2}\sqrt{3} = \frac{7}{2}\sqrt{3}$$

$$\mathbf{c} \quad f'(x) = \frac{\frac{1}{2 \cos x} \times (-2 \sin x) \times \sin x - \ln(2 \cos x) \times \cos x}{\sin^2 x} \\ = -\sec x - \frac{\cos x \ln(2 \cos x)}{\sin^2 x}$$

$$f'\left(\frac{\pi}{3}\right) = -2 - 0 = -2$$

$$\mathbf{d} \quad f'(x) = \\ 2 \sin x \cos x \times \cos^3 x + \sin^2 x \times 3 \cos^2 x \times (-\sin x) \\ = \sin x \cos^2 x(2 \cos^2 x - 3 \sin^2 x)$$

$$f'\left(\frac{\pi}{6}\right) = \frac{1}{2} \times \frac{3}{4} (2 \times \frac{3}{4} - 3 \times \frac{1}{4}) = \frac{9}{32}$$

$$9 \quad x = 0 \quad \therefore y = 3$$

$$\frac{dy}{dx} = 1 \times \cos 2x + x \times (-2 \sin 2x) \\ = \cos 2x - 2x \sin 2x$$

$$\text{grad} = 1$$

$$\therefore \text{grad of normal} = -1$$

$$\therefore y - 3 = -(x - 0)$$

$$[y = 3 - x]$$

$$10 \quad \mathbf{a} = \frac{\cos x \times (1 - \sin x) - (2 + \sin x) \times (-\cos x)}{(1 - \sin x)^2} \\ = \frac{3 \cos x}{(1 - \sin x)^2}$$

$$\mathbf{b} \quad \text{SP: } \frac{3 \cos x}{(1 - \sin x)^2} = 0 \\ \cos x = 0$$

$$x \neq \frac{\pi}{2} \quad \therefore x = \frac{3\pi}{2}$$

$$\therefore \left(\frac{3\pi}{2}, \frac{1}{2}\right)$$

$$\mathbf{c} \quad x = \frac{\pi}{6} \quad \therefore y = 5$$

$$\text{grad} = 6\sqrt{3}$$

$$y - 5 = 6\sqrt{3} \left(x - \frac{\pi}{6}\right)$$

$$y = 6\sqrt{3}x + 5 - \sqrt{3}\pi$$

$$11 \quad a \quad \frac{dy}{dx} = -e^{-x} \times \sin x + e^{-x} \times \cos x$$

$$= e^{-x}(\cos x - \sin x)$$

$$\frac{d^2y}{dx^2} = -e^{-x} \times (\cos x - \sin x)$$

$$+ e^{-x} \times (-\sin x - \cos x)$$

$$= -2e^{-x} \cos x$$

$$b \quad \text{SP: } e^{-x}(\cos x - \sin x) = 0$$

$$\cos x - \sin x = 0$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}, \frac{\pi}{4} - \pi = -\frac{3\pi}{4}, \frac{\pi}{4}$$

$$x = -\frac{3\pi}{4}: \frac{d^2y}{dx^2} = \sqrt{2}e^{\frac{3\pi}{4}} (> 0)$$

$$x = \frac{\pi}{4}: \frac{d^2y}{dx^2} = -\sqrt{2}e^{-\frac{\pi}{4}} (< 0)$$

$$\therefore \left(-\frac{3\pi}{4}, -\frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}\right), \text{ minimum}$$

$$\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}}\right), \text{ maximum}$$

$$12 \quad a \quad \frac{dy}{dx} = 1 \times \sec x + x \times \sec x \tan x$$

$$= \sec x(1 + x \tan x)$$

$$\text{SP: } \sec x(1 + x \tan x) = 0$$

no real values of  $x$  for which  $\sec x = 0$

$$\therefore 1 + x \tan x = 0$$

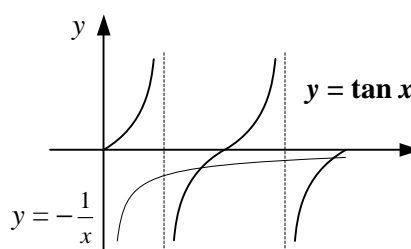
$$b \quad 1 + x \tan x = 0$$

$$\Rightarrow x \tan x = -1$$

$$\tan x = -\frac{1}{x}$$

$\therefore$   $x$ -coord of SP where curves

$$y = \tan x \text{ and } y = -\frac{1}{x} \text{ intersect}$$



intersect at 2 points  $\therefore$  2 SP in interval

$$13 \quad a \quad f'(x) = -\sin x \times \sin 2x + \cos x \times 2 \cos 2x$$

$$= 2 \cos x(1 - 2 \sin^2 x) - 2 \sin^2 x \cos x$$

$$= 2 \cos x(1 - 3 \sin^2 x)$$

$$b \quad \text{SP: } 2 \cos x(1 - 3 \sin^2 x) = 0$$

$$\cos x = 0 \text{ or } \sin x = \pm \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or } 0.615, \pi - 0.61548,$$

$$\pi + 0.61548, 2\pi - 0.61548$$

$$x = 0.615, \frac{\pi}{2}, 2.53, 3.76, \frac{3\pi}{2}, 5.67$$

$$c \quad \text{using graph, max. } f(x) \text{ when } \sin x = \frac{1}{\sqrt{3}}$$

$$\cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{3} = \frac{2}{3}$$

$$f(x) = \cos x \times 2 \sin x \cos x = 2 \sin x \cos^2 x$$

$$\therefore \text{max. } f(x) = 2 \times \frac{1}{\sqrt{3}} \times \frac{2}{3}$$

$$= \frac{4}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4}{9}\sqrt{3}$$

$$d \quad \text{period of } \cos x = 2\pi, \text{ period of } \sin 2x = \pi$$

$$\therefore \text{period of } f(x) = 2\pi$$

$\therefore$  values of  $f(x)$  in this interval are repeated

$$14 \quad a \quad x = 0 \therefore y = -2$$

$$\frac{dy}{dx} = -\operatorname{cosec} \left(x - \frac{\pi}{6}\right) \cot \left(x - \frac{\pi}{6}\right)$$

$$\text{grad} = -2\sqrt{3}$$

$$\therefore \text{grad of normal} = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{6}\sqrt{3}$$

$$\therefore y + 2 = \frac{1}{6}\sqrt{3}(x - 0)$$

$$[y = \frac{1}{6}\sqrt{3}x - 2]$$

$$b \quad x = \frac{\pi}{3} \therefore y = 2$$

$$\text{grad} = -2\sqrt{3}$$

$$\therefore y - 2 = -2\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$[6\sqrt{3}x + 3y - 6 - 2\sqrt{3}\pi = 0]$$

$$c \quad \frac{1}{6}\sqrt{3}x - 2 = 2 - 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$x - 4\sqrt{3} = 4\sqrt{3} - 12x + 4\pi$$

$$13x = 8\sqrt{3} + 4\pi$$

$$x = \frac{8\sqrt{3} + 4\pi}{13}$$