C3 DIFFERENTIATION — Answers - Worksheet G

1 **a** $\frac{dy}{dx} = 2x\times(2-x)^3 + x^2\times 3(2-x)^2\times(-1)$

$= x(2-x)^2(4-5x)$

grad = –1

$\therefore\ y - 1 = -(x-1)$ [$y = 2 - x$]

b grad of normal = 1

$\therefore\ y - 1 = x - 1$

$y = x$

when $x = 0$, $y = 0$ $\therefore$ passes through origin

2 **a** $\frac{dy}{dx} = \frac{1\times(2x+3) - x\times 2}{(2x+3)^2} = \frac{3}{(2x+3)^2}$

grad = 3

$\therefore\ y + 1 = 3(x+1)$ [$y = 3x + 2$]

b at (0, 0), grad = $\frac{1}{3}$

$\therefore$ grad of normal = –3

$\therefore\ y = -3x$

c $3x + 2 = -3x$

$x = -\frac{1}{3}$ $\therefore$ $(-\frac{1}{3}, 1)$

3 **a** $P(-3, 0)$, $Q(1, 0)$

b $\frac{dy}{dx} = 1\times(x-1)^3 + (x+3)\times 3(x-1)^2$

$= (x-1)^2[(x-1) + 3(x+3)]$

$= 4(x+2)(x-1)^2$

SP: $4(x+2)(x-1)^2 = 0$

$x = 1$ (at Q) or -2

$\therefore\ R(-2, -27)$

4 **a** $\frac{dy}{dx} = 1\times\sqrt{4x+1} + x\times\frac{1}{2}(4x+1)^{-\frac{1}{2}}\times 4$

$= (4x+1)^{-\frac{1}{2}}[(4x+1) + 2x] = \frac{6x+1}{\sqrt{4x+1}}$

b $\frac{6x+1}{\sqrt{4x+1}} - 5x\sqrt{4x+1} = 0$

$6x + 1 = 5x(4x+1)$

$20x^2 - x - 1 = 0$

$(5x+1)(4x-1) = 0$

$x = -\frac{1}{5}, \frac{1}{4}$

5 **a** at A, $y = 0$ $\therefore\ x = 1$

$\frac{dy}{dx} = \frac{2\times(x^2+3) - 2(x-1)\times 2x}{(x^2+3)^2}$

$= \frac{6 + 4x - 2x^2}{(x^2+3)^2}$

$\therefore$ grad = $\frac{1}{2}$ $\therefore$ grad of normal = –2

$\therefore\ y - 0 = -2(x-1)$

$y = 2 - 2x$

b SP: $\frac{6+4x-2x^2}{(x^2+3)^2} = 0$

$2(1+x)(3-x) = 0$

$x = -1, 3$

$\therefore\ (-1, -1), (3, \frac{1}{3})$

6 **a** $f'(x) = \frac{3}{2}x^{\frac{1}{2}}\times(x-3)^3 + x^{\frac{3}{2}}\times 3(x-3)^2$

$= \frac{3}{2}x^{\frac{1}{2}}(x-3)^2[(x-3) + 2x]$

$= \frac{3}{2}x^{\frac{1}{2}}(3x-3)(x-3)^2$

$= \frac{9}{2}x^{\frac{1}{2}}(x-1)(x-3)^2$ [$k = \frac{9}{2}$]

b SP: $\frac{9}{2}x^{\frac{1}{2}}(x-1)(x-3)^2 = 0$

$x > 0$ $\therefore\ x = 1, 3$

$\therefore$ (1, –8), (3, 0)

7 **a** $f'(x) = 1\times\sqrt{2x+12} + x\times\frac{1}{2}(2x+12)^{-\frac{1}{2}}\times 2$

$= (2x+12)^{-\frac{1}{2}}[(2x+12) + x]$

$= \frac{3x+12}{\sqrt{2x+12}}$

$f''(x) = \frac{3\times\sqrt{2x+12} - (3x+12)\times\frac{1}{2}(2x+12)^{-\frac{1}{2}}\times 2}{2x+12}$

$= \frac{3(2x+12) - (3x+12)}{(2x+12)^{\frac{3}{2}}} = \frac{3x+24}{(2x+12)^{\frac{3}{2}}}$

$= \frac{3(x+8)}{(2x+12)^{\frac{3}{2}}}$

b SP: $\frac{3x+12}{\sqrt{2x+12}} = 0$

$x = -4$

$\therefore\ (-4, -8)$

$f''(-4) = \frac{3}{2}$

$f''(-4) > 0$ $\therefore$ minimum