DIFFERENTIATION

Worksheet F

Given that $f(x) = \frac{x}{x+2}$, find f'(x)1

a using the product rule,

b using the quotient rule.

Differentiate each of the following with respect to x and simplify your answers. 2

$$\mathbf{a} = \frac{4x}{1-3x}$$

$$\mathbf{b} \quad \frac{\mathrm{e}^x}{x-4}$$

$$\mathbf{c} = \frac{x+1}{2x+3}$$

$$\mathbf{d} \quad \frac{\ln x}{2x}$$

$$e \frac{x}{2-x^2}$$

e
$$\frac{x}{2-x^2}$$
 f $\frac{\sqrt{x}}{3x+2}$ **g** $\frac{e^{2x}}{1-e^{2x}}$ **h** $\frac{2x+1}{\sqrt{x-3}}$

$$\mathbf{g} \quad \frac{\mathrm{e}^{2x}}{1-\mathrm{e}^{2x}}$$

$$\mathbf{h} \quad \frac{2x+1}{\sqrt{x-3}}$$

Find $\frac{dy}{dx}$, simplifying your answer in each case.

$$\mathbf{a} \quad y = \frac{x^2}{x+4}$$

b
$$y = \frac{\sqrt{x-4}}{2x^2}$$

$$\mathbf{c} \quad y = \frac{2e^x + 1}{1 - 3e^x}$$

d
$$y = \frac{1-x}{x^3+2}$$

$$\mathbf{e} \quad y = \frac{\ln(3x-1)}{x+2}$$

$$\mathbf{f}$$
 $y = \sqrt{\frac{x+1}{x+3}}$

Find the coordinates of any stationary points on each curve. 4

a
$$y = \frac{x^2}{3-x}$$

b
$$y = \frac{e^{4x}}{2x-1}$$

$$\mathbf{c} \quad y = \frac{x+5}{\sqrt{2x+1}}$$

$$\mathbf{d} \quad y = \frac{\ln 3x}{2x}$$

$$\mathbf{e} \quad y = \left(\frac{x+1}{x-2}\right)^2$$

f
$$y = \frac{x^2 - 3}{x + 2}$$

5 Find an equation for the tangent to each curve at the point on the curve with the given *x*-coordinate.

a
$$y = \frac{2x}{3-x}$$
, $x = 2$

$$x = 2$$

b
$$y = \frac{e^x + 3}{e^x + 1},$$
 $x = 0$

$$x = 0$$

$$\mathbf{c} \quad y = \frac{\sqrt{x}}{5 - x}, \qquad x = 4$$

$$x = 4$$

d
$$y = \frac{3x+4}{x^2+1}, \qquad x = -1$$

Find an equation for the normal to each curve at the point on the curve with the given x-coordinate. 6 Give your answers in the form ax + by + c = 0, where a, b and c are integers.

a
$$y = \frac{1-x}{3x+1},$$
 $x = 1$

$$x = 1$$

b
$$y = \frac{4x}{\sqrt{2-x}}, \qquad x = -2$$

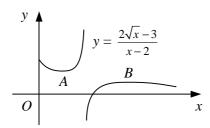
$$x = -2$$

c
$$y = \frac{\ln(2x-5)}{3x-5}$$
, $x = 3$

d
$$y = \frac{x}{x^3 - 4}$$
, $x = 2$

$$x = 2$$

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The diagram shows part of the curve $y = \frac{2\sqrt{x}-3}{x-2}$ which is stationary at the points A and B.

a Show that the x-coordinates of A and B satisfy the equation $x - 3\sqrt{x} + 2 = 0$.

b Hence, find the coordinates of *A* and *B*.