

C3 DIFFERENTIATION

Answers - Worksheet F

1 a $f(x) = x(x+2)^{-1}$

$$\begin{aligned} f'(x) &= 1 \times (x+2)^{-1} + x \times [-(x+2)^{-2}] \\ &= (x+2)^{-2}[(x+2) - x] \\ &= \frac{2}{(x+2)^2} \end{aligned}$$

b $f'(x) = \frac{1 \times (x+2) - x \times 1}{(x+2)^2}$

$$= \frac{2}{(x+2)^2}$$

2 a $= \frac{4 \times (1-3x) - 4x \times (-3)}{(1-3x)^2}$

$$= \frac{4}{(1-3x)^2}$$

c $= \frac{1 \times (2x+3) - (x+1) \times 2}{(2x+3)^2}$

$$= \frac{1}{(2x+3)^2}$$

e $= \frac{1 \times (2-x^2) - x \times (-2x)}{(2-x^2)^2}$

$$= \frac{2+x^2}{(2-x^2)^2}$$

g $= \frac{2e^{2x} \times (1-e^{2x}) - e^{2x} \times (-2e^{2x})}{(1-e^{2x})^2}$

$$= \frac{2e^{2x}}{(1-e^{2x})^2}$$

3 a $= \frac{2x \times (x+4) - x^2 \times 1}{(x+4)^2}$

$$= \frac{x^2 + 8x}{(x+4)^2}$$

c $= \frac{2e^x \times (1-3e^x) - (2e^x+1) \times (-3e^x)}{(1-3e^x)^2}$

$$= \frac{5e^x}{(1-3e^x)^2}$$

e $= \frac{\frac{3}{3x-1} \times (x+2) - \ln(3x-1) \times 1}{(x+2)^2}$

$$= \frac{3}{(3x-1)(x+2)} - \frac{\ln(3x-1)}{(x+2)^2}$$

b $= \frac{e^x \times (x-4) - e^x \times 1}{(x-4)^2}$

$$= \frac{e^x(x-5)}{(x-4)^2}$$

d $= \frac{\frac{1}{x} \times 2x - \ln x \times 2}{(2x)^2}$

$$= \frac{1 - \ln x}{2x^2}$$

f $= \frac{\frac{1}{2}x^{-\frac{1}{2}} \times (3x+2) - \sqrt{x} \times 3}{(3x+2)^2}$

$$= \frac{(3x+2) - 6x}{2\sqrt{x}(3x+2)^2} = \frac{2-3x}{2\sqrt{x}(3x+2)^2}$$

h $= \frac{2 \times \sqrt{x-3} - (2x+1) \times \frac{1}{2}(x-3)^{-\frac{1}{2}}}{x-3}$

$$= \frac{4(x-3) - (2x+1)}{2(x-3)^{\frac{3}{2}}} = \frac{2x-13}{2(x-3)^{\frac{3}{2}}}$$

b $= \frac{\frac{1}{2}(x-4)^{-\frac{1}{2}} \times 2x^2 - \sqrt{x-4} \times 4x}{(2x^2)^2}$

$$= \frac{x-4(x-4)}{4x^3\sqrt{x-4}} = \frac{16-3x}{4x^3\sqrt{x-4}}$$

d $= \frac{-1 \times (x^3+2) - (1-x) \times 3x^2}{(x^3+2)^2}$

$$= \frac{2x^3 - 3x^2 - 2}{(x^3+2)^2}$$

f $y = \frac{\sqrt{x+1}}{\sqrt{x+3}}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{1}{2}(x+1)^{-\frac{1}{2}} \times \sqrt{x+3} - \sqrt{x+1} \times \frac{1}{2}(x+3)^{-\frac{1}{2}}}{x+3} \\ &= \frac{(x+3) - (x+1)}{2(x+1)^{\frac{1}{2}}(x+3)^{\frac{3}{2}}} = \frac{1}{(x+1)^{\frac{1}{2}}(x+3)^{\frac{3}{2}}} \end{aligned}$$

$$4 \quad \mathbf{a} \quad \frac{dy}{dx} = \frac{2x \times (3-x) - x^2 \times (-1)}{(3-x)^2}$$

$$= \frac{6x - x^2}{(3-x)^2}$$

$$\text{SP: } \frac{x(6-x)}{(3-x)^2} = 0$$

$$x = 0, 6$$

$$\therefore (0, 0), (6, -12)$$

$$\mathbf{c} \quad \frac{dy}{dx} = \frac{1 \times \sqrt{2x+1} - (x+5) \times \frac{1}{2}(2x+1)^{-\frac{1}{2}} \times 2}{2x+1}$$

$$= \frac{(2x+1) - (x+5)}{(2x+1)^{\frac{3}{2}}} = \frac{x-4}{(2x+1)^{\frac{3}{2}}}$$

$$\text{SP: } \frac{x-4}{(2x+1)^{\frac{3}{2}}} = 0$$

$$x = 4$$

$$\therefore (4, 3)$$

$$\mathbf{e} \quad y = \frac{(x+1)^2}{(x-2)^2}$$

$$\frac{dy}{dx} = \frac{2(x+1) \times (x-2)^2 - (x+1)^2 \times 2(x-2)}{(x-2)^4}$$

$$= \frac{2(x+1)[(x-2) - (x+1)]}{(x-2)^3} = \frac{-6(x+1)}{(x-2)^3}$$

$$\text{SP: } \frac{-6(x+1)}{(x-2)^3} = 0$$

$$x = -1$$

$$\therefore (-1, 0)$$

$$5 \quad \mathbf{a} \quad x = 2 \quad \therefore y = 4$$

$$\frac{dy}{dx} = \frac{2 \times (3-x) - 2x \times (-1)}{(3-x)^2}$$

$$= \frac{6}{(3-x)^2}$$

$$\text{grad} = 6$$

$$\therefore y - 4 = 6(x - 2)$$

$$[y = 6x - 8]$$

$$\mathbf{c} \quad x = 4 \quad \therefore y = 2$$

$$\frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}} \times (5-x) - \sqrt{x} \times (-1)}{(5-x)^2}$$

$$= \frac{(5-x) + 2x}{2\sqrt{x}(5-x)^2} = \frac{5+x}{2\sqrt{x}(5-x)^2}$$

$$\text{grad} = \frac{9}{4}$$

$$\therefore y - 2 = \frac{9}{4}(x - 4)$$

$$[y = \frac{9}{4}x - 7]$$

$$\mathbf{b} \quad \frac{dy}{dx} = \frac{4e^{4x} \times (2x-1) - e^{4x} \times 2}{(2x-1)^2}$$

$$= \frac{2e^{4x}(4x-3)}{(2x-1)^2}$$

$$\text{SP: } \frac{2e^{4x}(4x-3)}{(2x-1)^2} = 0$$

$$x = \frac{3}{4}$$

$$\therefore \left(\frac{3}{4}, 2e^3\right)$$

$$\mathbf{d} \quad \frac{dy}{dx} = \frac{\frac{1}{x} \times 2x - \ln 3x \times 2}{(2x)^2}$$

$$= \frac{1 - \ln 3x}{2x^2}$$

$$\text{SP: } \frac{1 - \ln 3x}{2x^2} = 0$$

$$x = \frac{1}{3}e$$

$$\therefore \left(\frac{1}{3}e, \frac{3}{2}e^{-1}\right)$$

$$\mathbf{f} \quad \frac{dy}{dx} = \frac{2x \times (x+2) - (x^2-3) \times 1}{(x+2)^2}$$

$$= \frac{x^2 + 4x + 3}{(x+2)^2}$$

$$\text{SP: } \frac{(x+1)(x+3)}{(x+2)^2} = 0$$

$$x = -3, -1$$

$$\therefore (-3, -6), (-1, -2)$$

$$\mathbf{b} \quad x = 0 \quad \therefore y = 2$$

$$\frac{dy}{dx} = \frac{e^x \times (e^x + 1) - (e^x + 3) \times e^x}{(e^x + 1)^2}$$

$$= \frac{-2e^x}{(e^x + 1)^2}$$

$$\text{grad} = -\frac{1}{2}$$

$$\therefore y - 2 = -\frac{1}{2}(x - 0)$$

$$[y = 2 - \frac{1}{2}x]$$

$$\mathbf{d} \quad x = -1 \quad \therefore y = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{3 \times (x^2 + 1) - (3x + 4) \times 2x}{(x^2 + 1)^2}$$

$$= \frac{3 - 8x - 3x^2}{(x^2 + 1)^2}$$

$$\text{grad} = 2$$

$$\therefore y - \frac{1}{2} = 2(x + 1)$$

$$[y = 2x + \frac{5}{2}]$$

6 a $x = 1 \therefore y = 0$

$$\frac{dy}{dx} = \frac{-1 \times (3x+1) - (1-x) \times 3}{(3x+1)^2}$$

$$= \frac{-4}{(3x+1)^2}$$

$$\text{grad} = -\frac{1}{4}$$

$$\therefore \text{grad of normal} = 4$$

$$\therefore y - 0 = 4(x - 1)$$

$$4x - y - 4 = 0$$

c $x = 3 \therefore y = 0$

$$\frac{dy}{dx} = \frac{\frac{2}{2x-5} \times (3x-5) - \ln(2x-5) \times 3}{(3x-5)^2}$$

$$= \frac{2}{(2x-5)(3x-5)} - \frac{3 \ln(2x-5)}{(3x-5)^2}$$

$$\text{grad} = \frac{1}{2}$$

$$\therefore \text{grad of normal} = -2$$

$$\therefore y - 0 = -2(x - 3)$$

$$2x + y - 6 = 0$$

7 a $\frac{dy}{dx} = \frac{x^{-\frac{1}{2}} \times (x-2) - (2\sqrt{x}-3) \times 1}{(x-2)^2}$

$$= \frac{(x-2) - \sqrt{x}(2\sqrt{x}-3)}{\sqrt{x}(x-2)^2}$$

$$= \frac{-(x-3\sqrt{x}+2)}{\sqrt{x}(x-2)^2}$$

at A and B, $\frac{dy}{dx} = 0 \therefore x - 3\sqrt{x} + 2 = 0$

b $(\sqrt{x} - 1)(\sqrt{x} - 2) = 0$

$$\sqrt{x} = 1, 2$$

$$x = 1, 4$$

$$\therefore A(1, 1), B(4, \frac{1}{2})$$

b $x = -2 \therefore y = -4$

$$\frac{dy}{dx} = \frac{4 \times \sqrt{2-x} - 4x \times \frac{1}{2}(2-x)^{-\frac{1}{2}} \times (-1)}{2-x}$$

$$= \frac{4(2-x) + 2x}{(2-x)^{\frac{3}{2}}} = \frac{8-2x}{(2-x)^{\frac{3}{2}}}$$

$$\text{grad} = \frac{3}{2}$$

$$\therefore \text{grad of normal} = -\frac{2}{3}$$

$$\therefore y + 4 = -\frac{2}{3}(x + 2)$$

$$2x + 3y + 16 = 0$$

d $x = 2 \therefore y = \frac{1}{2}$

$$\frac{dy}{dx} = \frac{1 \times (x^3 - 4) - x \times 3x^2}{(x^3 - 4)^2}$$

$$= \frac{-4 - 2x^3}{(x^3 - 4)^2}$$

$$\text{grad} = -\frac{5}{4}$$

$$\therefore \text{grad of normal} = \frac{4}{5}$$

$$\therefore y - \frac{1}{2} = \frac{4}{5}(x - 2)$$

$$8x - 10y - 11 = 0$$