

C3 DIFFERENTIATION

Answers - Worksheet E

- 1** **a** $f(x) = x[x^3 + 3x^2(2) + 3x(2)^2 + 2^3]$
 $= x^4 + 6x^3 + 12x^2 + 8x$
 $f'(x) = 4x^3 + 18x^2 + 24x + 8$
- b** $f'(x) = 1 \times (x+2)^3 + x \times 3(x+2)^2$
 $= (x+2)^2[(x+2) + 3x]$
 $= 2(2x+1)(x+2)^2$
- 2** **a** $= 1 \times e^x + x \times e^x$
 $= e^x(1+x)$
- b** $= 1 \times (x+1)^5 + x \times 5(x+1)^4$
 $= (x+1)^4[(x+1) + 5x]$
 $= (6x+1)(x+1)^4$
- c** $= 1 \times \ln x + x \times \frac{1}{x}$
 $= \ln x + 1$
- d** $= 2x \times (x-1)^3 + x^2 \times 3(x-1)^2$
 $= x(x-1)^2[2(x-1) + 3x]$
 $= x(5x-2)(x-1)^2$
- e** $= 3x^2 \times \ln 2x + x^3 \times \frac{1}{x}$
 $= x^2(3 \ln 2x + 1)$
- f** $= 2x \times e^{-x} + x^2 \times (-e^{-x})$
 $= xe^{-x}(2-x)$
- g** $= 8x^3 \times (5+x)^3 + 2x^4 \times 3(5+x)^2$
 $= 2x^3(5+x)^2[4(5+x) + 3x]$
 $= 2x^3(20+7x)(5+x)^2$
- h** $= 2x \times (x-3)^4 + x^2 \times 4(x-3)^3$
 $= 2x(x-3)^3[(x-3) + 2x]$
 $= 6x(x-1)(x-3)^3$
- 3** **a** $= 1 \times (2x-1)^3 + x \times 3(2x-1)^2 \times 2$
 $= (2x-1)^2[(2x-1) + 6x]$
 $= (8x-1)(2x-1)^2$
- b** $= 12x^3 \times e^{2x+3} + 3x^4 \times e^{2x+3} \times 2$
 $= 6x^3e^{2x+3}(2+x)$
- c** $= 1 \times \sqrt{x-1} + x \times \frac{1}{2}(x-1)^{-\frac{1}{2}}$
 $= \frac{1}{2}(x-1)^{-\frac{1}{2}}[2(x-1) + x]$
 $= \frac{3x-2}{2\sqrt{x-1}}$
- d** $= 2x \times \ln(x+6) + x^2 \times \frac{1}{x+6}$
 $= 2x \ln(x+6) + \frac{x^2}{x+6}$
- e** $= 1 \times (1-5x)^4 + x \times 4(1-5x)^3 \times (-5)$
 $= (1-5x)^3[(1-5x) - 20x]$
 $= (1-25x)(1-5x)^3$
- f** $= 1 \times (x-3)^3 + (x+2) \times 3(x-3)^2$
 $= (x-3)^2[(x-3) + 3(x+2)]$
 $= (4x+3)(x-3)^2$
- g** $= \frac{4}{3}x^{\frac{1}{3}} \times e^{3x} + x^{\frac{4}{3}} \times 3e^{3x}$
 $= \frac{1}{3}x^{\frac{1}{3}}e^{3x}(4+9x)$
- h** $= 1 \times \ln(x^2-1) + (x+1) \times \frac{1}{x^2-1} \times 2x$
 $= \ln(x^2-1) + \frac{2x(x+1)}{(x+1)(x-1)}$
 $= \ln(x^2-1) + \frac{2x}{x-1}$
- i** $= 2x \times \sqrt{3x+1} + x^2 \times \frac{1}{2}(3x+1)^{-\frac{1}{2}} \times 3$
 $= \frac{1}{2}x(3x+1)^{-\frac{1}{2}}[4(3x+1) + 3x]$
 $= \frac{x(15x+4)}{2\sqrt{3x+1}}$

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Answers - Worksheet E page 2

- 4 a** $f'(x) = 4 \times e^{3x} + 4x \times 3e^{3x}$
 $= 4e^{3x}(1 + 3x)$
 $f'(0) = 4 \times 1 \times 1 = 4$
- c** $f'(x) = 5 \times \ln 3x + (5x - 4) \times \frac{1}{x}$
 $= 5 \ln 3x + 5 - \frac{4}{x}$
 $f'(\frac{1}{3}) = 0 + 5 - 12 = -7$
- 5 a** $\frac{dy}{dx} = 1 \times e^{2x} + x \times 2e^{2x}$
 SP: $e^{2x}(1 + 2x) = 0$
 $x = -\frac{1}{2}$
 $\therefore (-\frac{1}{2}, -\frac{1}{2}e^{-1})$
- c** $\frac{dy}{dx} = 2x \times (2x - 3)^4 + x^2 \times 4(2x - 3)^3 \times 2$
 $= 2x(2x - 3)^3[(2x - 3) + 4x]$
 SP: $6x(2x - 1)(2x - 3)^3 = 0$
 $x = 0, \frac{1}{2}, \frac{3}{2}$
 $\therefore (0, 0), (\frac{1}{2}, 4), (\frac{3}{2}, 0)$
- e** $\frac{dy}{dx} = 2x \times e^{-4x} + x^2 \times (-4e^{-4x})$
 SP: $2xe^{-4x}(1 - 2x) = 0$
 $x = \frac{1}{2}$
 $\therefore (\frac{1}{2}, 2 + \frac{1}{4}e^{-2})$
- 6 a** $x = 1 \therefore y = 1$
 $\frac{dy}{dx} = 1 \times (x - 2)^4 + x \times 4(x - 2)^3$
 $= (x - 2)^3[(x - 2) + 4x]$
 $= (5x - 2)(x - 2)^3$
 grad = -3
 $\therefore y - 1 = -3(x - 1)$
 $[y = 4 - 3x]$
- c** $x = \frac{1}{2} \therefore y = 0$
 $\frac{dy}{dx} = 4 \times \ln 2x + (4x - 1) \times \frac{1}{x}$
 $= 4 \ln 2x + 4 - \frac{1}{x}$
 grad = 2
 $\therefore y - 0 = 2(x - \frac{1}{2})$
 $[y = 2x - 1]$
- b** $f'(x) = 2 \times (x^2 + 2)^3 + 2x \times 3(x^2 + 2)^2 \times 2x$
 $= 2(x^2 + 2)^2[(x^2 + 2) + 6x^2]$
 $= 2(7x^2 + 2)(x^2 + 2)^2$
 $f'(-1) = 2 \times 9 \times 9 = 162$
- d** $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \times (1 - 2x)^3 + x^{\frac{1}{2}} \times 3(1 - 2x)^2 \times (-2)$
 $= \frac{1}{2}x^{-\frac{1}{2}}(1 - 2x)^2[(1 - 2x) - 12x]$
 $= \frac{1}{2}x^{-\frac{1}{2}}(1 - 14x)(1 - 2x)^2$
 $f'(\frac{1}{4}) = \frac{1}{2} \times 2 \times (-\frac{5}{2}) \times \frac{1}{4} = -\frac{5}{8}$
- b** $\frac{dy}{dx} = 1 \times (x - 4)^3 + x \times 3(x - 4)^2$
 $= (x - 4)^2[(x - 4) + 3x]$
 SP: $4(x - 1)(x - 4)^2 = 0$
 $x = 1, 4$
 $\therefore (1, -27), (4, 0)$
- d** $\frac{dy}{dx} = 1 \times \sqrt{x+12} + x \times \frac{1}{2}(x+12)^{-\frac{1}{2}}$
 $= \frac{1}{2}(x+12)^{-\frac{1}{2}}[2(x+12) + x]$
 SP: $\frac{3(x+8)}{2\sqrt{x+12}} = 0$
 $x = -8$
 $\therefore (-8, -16)$
- f** $\frac{dy}{dx} = -3 \times (3 - x)^3 + (1 - 3x) \times 3(3 - x)^2 \times (-1)$
 $= -3(3 - x)^2[(3 - x) + (1 - 3x)]$
 SP: $-12(1 - x)(3 - x)^2 = 0$
 $x = 1, 3$
 $\therefore (1, -16), (3, 0)$
- b** $x = 1 \therefore y = 3e$
 $\frac{dy}{dx} = 6x \times e^x + 3x^2 \times e^x$
 $= 3xe^x(2 + x)$
 grad = $9e$
 $\therefore y - 3e = 9e(x - 1)$
 $[y = 3e(3x - 2)]$
- d** $x = -2 \therefore y = 8$
 $\frac{dy}{dx} = 2x \times \sqrt{x+6} + x^2 \times \frac{1}{2}(x+6)^{-\frac{1}{2}}$
 $= \frac{1}{2}x(x+6)^{-\frac{1}{2}}[4(x+6) + x] = \frac{x(5x+24)}{2\sqrt{x+6}}$
 grad = -7
 $\therefore y - 8 = -7(x + 2)$
 $[y = -7x - 6]$

7 a $x = 1 \therefore y = 1$

$$\frac{dy}{dx} = 2x \times (2-x)^3 + x^2 \times 3(2-x)^2 \times (-1)$$

$$= x(2-x)^2[2(2-x) - 3x]$$

$$= x(4-5x)(2-x)^2$$

$$\text{grad} = -1$$

$$\therefore \text{grad of normal} = 1$$

$$\therefore y - 1 = 1(x - 1)$$

$$x - y = 0$$

c $x = 0 \therefore y = -1$

$$\frac{dy}{dx} = 2x \times e^{3x} + (x^2 - 1) \times 3e^{3x}$$

$$= e^{3x}[2x + 3(x^2 - 1)]$$

$$= e^{3x}(3x^2 + 2x - 3)$$

$$\text{grad} = -3$$

$$\therefore \text{grad of normal} = \frac{1}{3}$$

$$\therefore y + 1 = \frac{1}{3}(x - 0)$$

$$x - 3y - 3 = 0$$

b $x = 2 \therefore y = 0$

$$\frac{dy}{dx} = 1 \times \ln(3x - 5) + x \times \frac{1}{3x - 5} \times 3$$

$$= \ln(3x - 5) + \frac{3x}{3x - 5}$$

$$\text{grad} = 6$$

$$\therefore \text{grad of normal} = -\frac{1}{6}$$

$$\therefore y - 0 = -\frac{1}{6}(x - 2)$$

$$x + 6y - 2 = 0$$

d $x = 8 \therefore y = 16$

$$\frac{dy}{dx} = 1 \times \sqrt{x-4} + x \times \frac{1}{2}(x-4)^{-\frac{1}{2}}$$

$$= \frac{1}{2}(x-4)^{-\frac{1}{2}}[2(x-4) + x]$$

$$= \frac{3x-8}{2\sqrt{x-4}}$$

$$\text{grad} = 4$$

$$\therefore \text{grad of normal} = -\frac{1}{4}$$

$$\therefore y - 16 = -\frac{1}{4}(x - 8)$$

$$x + 4y - 72 = 0$$

8 a $x = 1 \therefore y = e$

$$\frac{dy}{dx} = 1 \times e^{x^2} + x \times e^{x^2} \times 2x$$

$$= e^{x^2}(1 + 2x^2)$$

$$\text{grad} = 3e$$

$$\therefore y - e = 3e(x - 1)$$

$$[y = e(3x - 2)]$$

b $x = 0 \Rightarrow y = -2e$

$$y = 0 \Rightarrow x = \frac{2}{3}$$

$$\therefore \text{area} = \frac{1}{2} \times 2e \times \frac{2}{3} = \frac{2}{3}e$$