

# C3 DIFFERENTIATION

## Worksheet D

1 Find an equation for the tangent to the curve with equation  $y = x^2 + \ln(4x - 1)$  at the point on the curve where  $x = \frac{1}{2}$ .

2 A curve has the equation  $y = \sqrt{8 - e^{2x}}$ .

The point  $P$  on the curve has  $y$ -coordinate 2.

a Find the  $x$ -coordinate of  $P$ .

b Show that the tangent to the curve at  $P$  has equation

$$2x + y = 2 + \ln 4.$$

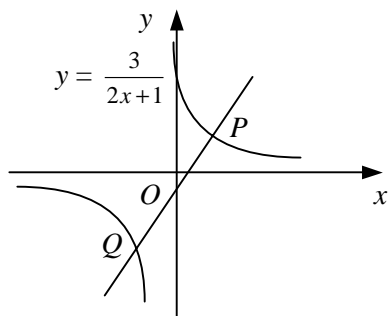
3 A curve has the equation  $y = 2x + 1 + \ln(4 - 2x)$ ,  $x < 2$ .

a Find and simplify expressions for  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ .

b Find the coordinates of the stationary point of the curve.

c Determine the nature of this stationary point.

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The diagram shows the curve with equation  $y = \frac{3}{2x+1}$ .

a Find an equation for the normal to the curve at the point  $P(1, 1)$ .

The normal to the curve at  $P$  intersects the curve again at the point  $Q$ .

b Find the exact coordinates of  $Q$ .

5 A quantity  $N$  is increasing such that at time  $t$  seconds,

$$N = ae^{kt}.$$

Given that at time  $t = 0$ ,  $N = 20$  and that at time  $t = 8$ ,  $N = 60$ , find

a the values of the constants  $a$  and  $k$ ,

b the value of  $N$  when  $t = 12$ ,

c the rate at which  $N$  is increasing when  $t = 12$ .

6

$$f(x) \equiv (5 - 2x^2)^3.$$

a Find  $f'(x)$ .

b Find the coordinates of the stationary points of the curve  $y = f(x)$ .

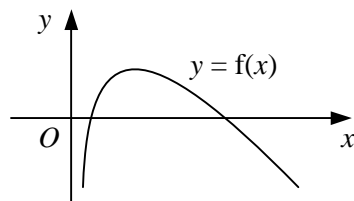
c Find the equation for the tangent to the curve  $y = f(x)$  at the point with  $x$ -coordinate  $\frac{3}{2}$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

## C3 DIFFERENTIATION

## Worksheet D continued

- 7 A curve has the equation  $y = 4x - \frac{1}{2}e^{2x}$ .
- Find the coordinates of the stationary point of the curve, giving your answers in terms of natural logarithms.
  - Determine the nature of the stationary point.

8



The diagram shows the curve  $y = f(x)$  where  $f(x) = 3 \ln 5x - 2x$ ,  $x > 0$ .

- Find  $f'(x)$ .
  - Find the  $x$ -coordinate of the point on the curve at which the gradient of the normal to the curve is  $-\frac{1}{4}$ .
  - Find the coordinates of the maximum turning point of the curve.
  - Write down the set of values of  $x$  for which  $f(x)$  is a decreasing function.
- 9 The curve  $C$  has the equation  $y = \sqrt{x^2 + 3}$ .
- Find an equation for the tangent to  $C$  at the point  $A(-1, 2)$ .
  - Find an equation for the normal to  $C$  at the point  $B(1, 2)$ .
  - Find the  $x$ -coordinate of the point where the tangent to  $C$  at  $A$  meets the normal to  $C$  at  $B$ .
- 10 A bucket of hot water is placed outside and allowed to cool. The surface temperature of the water,  $T$  °C, after  $t$  minutes is given by
- $$T = 20 + 60e^{-kt},$$
- where  $k$  is a positive constant.
- State the initial surface temperature of the water.
  - State, with a reason, the air temperature around the bucket.
- Given that  $T = 30$  when  $t = 25$ ,
- find the value of  $k$ ,
  - find the rate at which the surface temperature of the water is decreasing when  $t = 40$ .

11

$$f(x) \equiv x^2 - 7x + 4 \ln\left(\frac{x}{2}\right), \quad x > 0.$$

- Solve the equation  $f'(x) = 0$ , giving your answers correct to 2 decimal places.
- Find an equation for the tangent to the curve  $y = f(x)$  at the point on the curve where  $x = 2$ .

12

A curve has the equation  $y = x^2 - \frac{8}{x-1}$ .

- Show that the  $x$ -coordinate of any stationary point of the curve satisfies the equation  $x^3 - 2x^2 + x + 4 = 0$ .
- Hence, show that the curve has exactly one stationary point and find its coordinates.
- Determine the nature of this stationary point.