C3 DIFFERENTIATION — Answers - Worksheet C

1

a $= 5(x+3)^4$

b $= 3(2x-1)^2 \times 2$
$= 6(2x-1)^2$

c $= 7(8-x)^6 \times (-1)$
$= -7(8-x)^6$

d $= 12(3x+4)^5 \times 3$
$= 36(3x+4)^5$

e $= 4(6-5x)^3 \times (-5)$
$= -20(6-5x)^3$

f $= -(x-2)^{-2}$

g $= -12(2x+3)^{-4} \times 2$
$= -24(2x+3)^{-4}$

h $= -2(7-3x)^{-3} \times (-3)$

h $= 6(7-3x)^{-3}$

2

a $= 6e^{3t}$

b $= \frac{1}{2}(4t-1)^{-\frac{1}{2}} \times 4$
$= 2(4t-1)^{-\frac{1}{2}}$

c $= \dfrac{5}{t}$

d $= \frac{3}{2}(8-3t)^{\frac{1}{2}} \times (-3)$
$= -\frac{9}{2}(8-3t)^{\frac{1}{2}}$

e $= \dfrac{3}{6t+1} \times 6$
$= \dfrac{18}{6t+1}$

f $= \frac{1}{2}e^{5t+4} \times 5$
$= \frac{5}{2}e^{5t+4}$

g $= \dfrac{d}{dx}[6(2t-5)^{-\frac{1}{3}}]$
$= -2(2t-5)^{-\frac{4}{3}} \times 2$
$= -4(2t-5)^{-\frac{4}{3}}$

h $= \dfrac{2}{3-\frac{1}{4}t} \times (-\frac{1}{4})$
$= \dfrac{2}{t-12}$

3

a $\dfrac{dy}{dx} = 4(3x-1)^3 \times 3$
$= 12(3x-1)^3$
$\dfrac{d^2y}{dx^2} = 36(3x-1)^2 \times 3$
$= 108(3x-1)^2$

b $\dfrac{dy}{dx} = \dfrac{4}{1+2x} \times 2$
$= 8(1+2x)^{-1}$
$\dfrac{d^2y}{dx^2} = -8(1+2x)^{-2} \times 2$
$= \dfrac{-16}{(1+2x)^2}$

c $\dfrac{dy}{dx} = \frac{1}{2}(5-2x)^{-\frac{1}{2}} \times (-2)$
$= -(5-2x)^{-\frac{1}{2}}$
$\dfrac{d^2y}{dx^2} = \frac{1}{2}(5-2x)^{-\frac{3}{2}} \times (-2)$
$= -(5-2x)^{-\frac{3}{2}}$

4

a $f'(x) = 2x - \dfrac{6}{x}$
$f'(3) = 6 - 2 = 4$

b $f'(x) = 2 - e^{x-2}$
$f'(2) = 2 - 1 = 1$

c $f'(x) = 4(2-5x)^3 \times (-5) = -20(2-5x)^3$
$f'(\frac{1}{2}) = -20 \times (-\frac{1}{8}) = \frac{5}{2}$

d $f'(x) = -4(x+5)^{-2}$
$f'(-1) = -4 \times \frac{1}{16} = -\frac{1}{4}$

5

a $f'(x) = 2(3x+15)^{-\frac{1}{2}} \times 3 = 2$
$\dfrac{6}{\sqrt{3x+15}} = 2$
$\sqrt{3x+15} = 3$
$3x + 15 = 9$
$x = -2$

b $f'(x) = 2x - \dfrac{1}{x-2} = 5$
$2x(x-2) - 1 = 5(x-2)$
$2x^2 - 9x + 9 = 0$
$(2x-3)(x-3) = 0$
for real $f(x)$, $x > 2$ $\therefore x = 3$

6

a $= 3(x^2-4)^2 \times 2x$
$= 6x(x^2-4)^2$

b $= 12(3x^2+1)^5 \times 6x$
$= 72x(3x^2+1)^5$

c $= \dfrac{1}{3+2x^2} \times 4x$
$= \dfrac{4x}{3+2x^2}$

d $= \dfrac{d}{dx}[(4-x^2)^3]$
$= 3(4-x^2)^2 \times (-2x)$
$= -6x(4-x^2)^2$

e $= \dfrac{d}{dx}[(\frac{1}{2}x^4+3)^8]$
$= 8(\frac{1}{2}x^4+3)^7 \times 2x^3$
$= 16x^3(\frac{1}{2}x^4+3)^7$

f $= \dfrac{d}{dx}[(3-x^2)^{-\frac{1}{2}}]$
$= -\frac{1}{2}(3-x^2)^{-\frac{3}{2}} \times (-2x)$
$= x(3-x^2)^{-\frac{3}{2}}$

g $= 7e^{x^2} \times 2x$
$= 14xe^{x^2}$

h $= 4(1-5x+x^3)^3 \times (-5+3x^2)$
$= 4(3x^2-5)(1-5x+x^3)^3$

i $= \frac{3}{4-\sqrt{x}} \times (-\frac{1}{2}x^{-\frac{1}{2}})$
$= \frac{3}{2x-8\sqrt{x}}$

j $= 7(e^{4x}+2)^6 \times 4e^{4x}$
$= 28e^{4x}(e^{4x}+2)^6$

k $= -(5+4\sqrt{x})^{-2} \times 2x^{-\frac{1}{2}}$
$= \frac{-2}{\sqrt{x}(5+4\sqrt{x})^2}$

l $= 5(\frac{2}{x}-x)^4 \times (-2x^{-2}-1)$
$= -5(\frac{2}{x^2}+1)(\frac{2}{x}-x)^4$

7 **a** $\frac{dy}{dx} = 5(2x-3)^4 \times 2$
SP: $10(2x-3)^4 = 0$
$x = \frac{3}{2}$
$\therefore\ (\frac{3}{2}, 0)$

b $\frac{dy}{dx} = 3(x^2-4)^2 \times 2x$
SP: $6x(x^2-4)^2 = 0$
$x = 0$ or $x^2 = 4$
$x = 0, \pm 2$
$\therefore\ (-2, 0), (0, -64), (2, 0)$

c $\frac{dy}{dx} = 8 - 2e^{2x}$
SP: $8 - 2e^{2x} = 0$
$e^{2x} = 4$
$x = \frac{1}{2}\ln 4 = \ln 2$
$\therefore\ (\ln 2, 8\ln 2 - 4)$

d $\frac{dy}{dx} = \frac{1}{2}(1+2x^2)^{-\frac{1}{2}} \times 4x$
SP: $\frac{2x}{\sqrt{1+2x^2}} = 0$
$x = 0$
$\therefore\ (0, 1)$

e $\frac{dy}{dx} = \frac{2}{x-x^2} \times (1-2x)$
SP: $\frac{2(1-2x)}{x-x^2} = 0$
$x = \frac{1}{2}$
$\therefore\ (\frac{1}{2}, -4\ln 2)$

f $\frac{dy}{dx} = 4 - (x-3)^{-2}$
SP: $4 - \frac{1}{(x-3)^2} = 0$
$(x-3)^2 = \frac{1}{4},\ x - 3 = \pm\frac{1}{2}$
$x = \frac{5}{2}, \frac{7}{2}$
$\therefore\ (\frac{5}{2}, 8), (\frac{7}{2}, 16)$

8 **a** $x = 2 \ \therefore\ y = 1$
$\frac{dy}{dx} = 4(3x-7)^3 \times 3 = 12(3x-7)^3$
grad $= -12$
$\therefore\ y - 1 = -12(x-2)$
[$y = 25 - 12x$]

b $x = 0 \ \therefore\ y = 2$
$\frac{dy}{dx} = \frac{1}{1+4x} \times 4 = \frac{4}{1+4x}$
grad $= 4$
$\therefore\ y = 4x + 2$

c $x = 1 \ \therefore\ y = 3$
$\frac{dy}{dx} = -9(x^2+2)^{-2} \times 2x = -18x(x^2+2)^{-2}$
grad $= -2$
$\therefore\ y - 3 = -2(x-1)$
[$y = 5 - 2x$]

d $x = \frac{1}{4} \ \therefore\ y = \frac{1}{2}$
$\frac{dy}{dx} = \frac{1}{2}(5x-1)^{-\frac{1}{2}} \times 5 = \frac{5}{2}(5x-1)^{-\frac{1}{2}}$
grad $= 5$
$\therefore\ y - \frac{1}{2} = 5(x - \frac{1}{4})$
[$y = 5x - \frac{3}{4}$]

9 **a** $x = -2 \ \therefore\ y = -9$
$\frac{dy}{dx} = e^{4-x^2} \times (-2x) = -2x\,e^{4-x^2}$
grad $= 4 \ \therefore$ grad of normal $= -\frac{1}{4}$
$\therefore\ y + 9 = -\frac{1}{4}(x+2)$
[$y = -\frac{1}{4}x - \frac{19}{2}$]

b $x = \frac{1}{2} \ \therefore\ y = \frac{1}{8}$
$\frac{dy}{dx} = 3(1-2x^2)^2 \times (-4x) = -12x(1-2x^2)^2$
grad $= -\frac{3}{2} \ \therefore$ grad of normal $= \frac{2}{3}$
$\therefore\ y - \frac{1}{8} = \frac{2}{3}(x - \frac{1}{2})$
[$16x - 24y - 5 = 0$]

c $x = 1 \ \therefore\ y = \frac{1}{2}$
$\frac{dy}{dx} = -(2-\ln x)^{-2} \times (-\frac{1}{x}) = \frac{1}{x(2-\ln x)^2}$
grad $= \frac{1}{4} \ \therefore$ grad of normal $= -4$
$\therefore\ y - \frac{1}{2} = -4(x-1)$
[$y = \frac{9}{2} - 4x$]

d $x = 3 \ \therefore\ y = 6e$
$\frac{dy}{dx} = 2e^{\frac{x}{3}}$
grad $= 2e \ \therefore$ grad of normal $= -\frac{1}{2e}$
$\therefore\ y - 6e = -\frac{1}{2e}(x-3)$
[$x + 2ey - 12e^2 - 3 = 0$]