1 Either prove or disprove each of the following statements.

(i) 'If m and n are consecutive odd numbers, then at least one of m and n is a prime number.' **[2]**

(ii) 'If m and n are consecutive even numbers, then mn is divisible by 8.' **[2]**

2 **(i)** Disprove the following statement:

$3^n + 2$ is prime for all integers $n \geqslant 0$. **[2]**

(ii) Prove that no number of the form 3^n (where n is a positive integer) has 5 as its final digit. **[2]**

3 **(i)** Factorise fully $n^3 - n$. **[2]**

(ii) Hence prove that, if n is an integer, $n^3 - n$ is divisible by 6. **[2]**

4 Prove or disprove the following statement:

'No cube of an integer has 2 as its units digit.' **[2]**

5 Use the triangle in Fig. 4 to prove that $\sin^2\theta + \cos^2\theta = 1$. For what values of θ is this proof valid? **[3]**

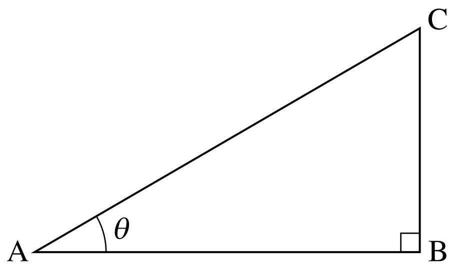

Fig. 4

6 **(i)** Multiply out $(3^n + 1)(3^n - 1)$. **[1]**

(ii) Hence prove that if n is a positive integer then $3^{2n} - 1$ is divisible by 8. **[3]**

7 State whether the following statements are true or false; if false, provide a counter-example.

(i) If a is rational and b is rational, then $a + b$ is rational.

(ii) If a is rational and b is irrational, then $a + b$ is irrational.

(iii) If a is irrational and b is irrational, then $a + b$ is irrational. **[3]**

8 **(i)** Disprove the following statement.

$$\text{‘If } p > q\text{, then } \frac{1}{p} < \frac{1}{q}\text{.’}$$ **[2]**

(ii) State a condition on p and q so that the statement is true. **[1]**

9 **(i)** Show that

(*A*) $(x - y)(x^2 + xy + y^2) = x^3 - y^3$,

(*B*) $(x + \frac{1}{2}y)^2 + \frac{3}{4}y^2 = x^2 + xy + y^2$. **[4]**

(ii) Hence prove that, for all real numbers x and y, if $x > y$ then $x^3 > y^3$. **[3]**

10 **(i)** Verify the following statement:

'$2^p - 1$ is a prime number for all prime numbers p less than 11'. [2]

(ii) Calculate 23×89, and hence disprove this statement:

'$2^p - 1$ is a prime number for all prime numbers p'. [2]

11 Use the method of exhaustion to prove the following result.

No 1- or 2-digit perfect square ends in 2, 3, 7 or 8

State a generalisation of this result. [3]

12 Prove that the following statement is false.

For all integers n greater than or equal to 1, $n^2 + 3n + 1$ is a prime number. [2]

13 Positive integers a, b and c are said to form a Pythagorean triple if $a^2 + b^2 = c^2$.

(i) Given that t is an integer greater than 1, show that $2t$, $t^2 - 1$ and $t^2 + 1$ form a Pythagorean triple. [3]

(ii) The two smallest integers of a Pythagorean triple are 20 and 21. Find the third integer.

Use this triple to show that not all Pythagorean triples can be expressed in the form $2t$, $t^2 - 1$ and $t^2 + 1$. [3]