1	**(i)**		False e.g. neither 25 and 27 are prime as 25 is div by 5 and 27 by 3	B1 B1 **[2]**	correct counter-example identified justified correctly	Need not explicitly say 'false'
1	**(ii)**		True: one has factor of 2, the other 4, so product must have factor of 8.	B2 **[2]**	or algebraic proofs: e.g. $2n(2n+2) = 4n(n+1) = 4\times$even$\times$odd no so div by 8	B1 for stating with justification div by 4 e.g. both even, or from $4(n^2 + n)$ or $4pq$

Question			Answer	Marks	Guidance	
2	**(i)**		$3^5 + 2 = 245$ [which is not prime]	M1 A1 **[2]**	Attempt to find counter-example correct counter-example identified	If A0, allow M1 for $3^n + 2$ correctly evaluated for 3 values of n
2	**(ii)**		$(3^0 = 1)$, $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, … so units digits cycle through 1, 3, 9, 7, 1, 3, … so cannot be a '5'. **OR** 3^n is not divisible by 5 all numbers ending in '5' are divisible by 5. so its last digit cannot be a '5'	M1 A1 B1 B1 **[2]**	Evaluate 3^n for $n = 0$ to 4 or 1 to 5 must state conclusion for B2	allow just final digit written

3	**(i)**		$n^3 - n = n(n^2 - 1)$ $= n(n - 1)(n + 1)$	B1 B1 **[2]**	two correct factors
3	**(ii)**		$n - 1$, n and $n + 1$ are consecutive integers so at least one is even, and one is div by 3 [$\Rightarrow$ $n^3 - n$ is div by 6]	B1 B1 **[2]**	

4			Cubes are 1, 8, 27, 64, 125, 216, 343, 512 [so false as] $8^3 = 512$	M1 A1 **[2]**	Attempt to find counter example counter-example identified (e.g. underlining, circling) [counter-examples all have 8 as units digit]	if no counter-example found, award M1 if at least 3 cubes are calculated. condone not explicitly stating statement is false

5 sin θ = BC/AC, cos θ = AB/AC $AB^2 + BC^2 = AC^2$ $\Rightarrow$ $(AB/AC)^2 + (BC/AC)^2 = 1$ $\Rightarrow$ $\cos^2\theta + \sin^2\theta = 1$ Valid for (0° <) θ < 90°	M1 A1 B1 [3]	or a/b, c/b condone taking AC = 1 Must use Pythagoras allow ≤, or ‘between 0 and 90’ or < 90 allow < π/2 or ‘acute’	allow o/h, a/h etc if clearly marked on triangle. but must be stated arguing backwards unless ⇔ used A0

6 (i) $(3^n + 1)(3^n - 1) = (3^n)^2 - 1$ or $3^{2n} - 1$	B1 [1]	mark final answer	or $9^n - 1$; penalise 3^{n^2} if it looks like 3 to the power n^2.
(ii) 3^n is odd $\Rightarrow 3^n + 1$ and $3^n - 1$ both even As consecutive even nos, one must be divisible by 4, so product is divisible by 8.	M1 M1 A1 [3]	3^n is odd $\Rightarrow 3^n + 1$ and $3^n - 1$ both even completion	Induction: If true for n, $3^{2n} - 1 = 8k$, so $3^{2n} = 1 + 8k$, M1 $3^{2(n+1)} - 1 = 9\times(8k + 1) - 1 = 72k + 8 = 8(9k + 1)$ so div by 8. A1 When $n = 1$, $3^2 - 1 = 8$ div by 8, true A1(or similar with 9^n)

7 (A) Tr e , (B) True , (C) False Counterexample, e.g. $\sqrt{2} + (-\sqrt{2}) = 0$	B2,1,0 B1 [3]		

8(i) e.g $p = 1$ and $q = -2$ $p > q$ but $1/p = 1 > 1/q = -½$	M1 E1 [2]	stating values of p, q with $p \geq 0$ and $q \leq 0$ (but not $p = q = 0$) showing that $1/p > 1/q$ - if 0 used, must state that 1/0 is undefined or infinite
(ii) Both p and q positive (or negative)	B1 [1]	or $q > 0$, 'positive integers'

9(i) (A) $(x-y)(x^2+xy+y^2)$ $= x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3$ $= x^3 - y^3$ * (B) $x + \frac{1}{2}y)^2 + \frac{3}{4}y^2$ $= x^2 + xy + \frac{1}{4}y^2 + \frac{3}{4}y^2$ $= x^2 + xy + y^2$	M1 E1 M1 E1 [4]	expanding - allow tabulation www $(x + \frac{1}{2}y)^2 = x^2 + \frac{1}{2}xy + \frac{1}{2}xy + \frac{1}{4}y^2$ o.e. cao www
(ii) $x^3 - y^3 = (x-y)[(x + \frac{1}{2}y)^2 + \frac{3}{4}y^2]$ $(x + \frac{1}{2}y)^2 + \frac{3}{4}y^2 > 0$ [as squares ≥ 0] $\Rightarrow$ if $x - y > 0$ then $x^3 - y^3 > 0$ $\Rightarrow$ if $x > y$ then $x^3 > y^3$ *	M1 M1 E1 [3]	substituting results of (i)

10(i) $p = 2$, $2^p - 1 = 3$, prime $p = 3$, $2^p - 1 = 7$, prime $p = 5$, $2^p - 1 = 31$, prime $p = 7$, $2^p - 1 = 127$, prime	M1 E1 [2]	Testing at least one prime testing all 4 primes (correctly) Must comment on answers being prime (allow ticks) Testing $p = 1$ is E0
(ii) $23 \times 89 = 2047 = 2^{11} - 1$ 11 is prime, 2047 is not So statement is false.	M1 E1 [2]	$2^{11} - 1$ must state or imply that 11 is prime ($p = 11$ is sufficient)

11 Perfect squares are 0, 1, 4, 9, 16, 25, 36, 49, 64, 81 none of which end in a 2, 3, 7 or 8. Generalisation: no perfect squares end in a 2, 3, 7 or 8.	 M1 E1 B1 [3]	 Listing all 1- and 2- digit squares. Condone absence of 0^2, and listing squares of 2 digit nos (i.e. $0^2 - 19^2$) For extending result to include further square numbers.

12 $n = 1$, $n^2 + 3n + 1 = 5$ prime $n = 2$, $n^2 + 3n + 1 = 11$ prime $n = 3$, $n^2 + 3n + 1 = 19$ prime $n = 4$, $n^2 + 3n + 1 = 29$ prime $n = 5$, $n^2 + 3n + 1 = 41$ prime $n = 6$, $n^2 + 3n + 1 = 55$ not prime so statement is false	M1 E1 [2]	One or more trials shown finding a counter-example – must state that it is not prime.

13(i) $a^2 + b^2 = (2t)^2 + (t^2 - 1)^2$ $= 4t^2 + t^4 - 2t^2 + 1$ $= t^4 + 2t^2 + 1$ $= (t^2 + 1)^2 = c^2$ **(ii)** $c = \sqrt{(20^2 + 21^2)} = 29$ For example: $2t = 20 \Rightarrow t = 10$ $\Rightarrow$ $t^2 - 1 = 99$ which is not consistent with 21	M1 M1 E1 B1 M1 E1 [6]	substituting for a, b and c in terms of t Expanding brackets correctly www Attempt to find t Any valid argument or E2 'none of 20, 21, 29 differ by two'.