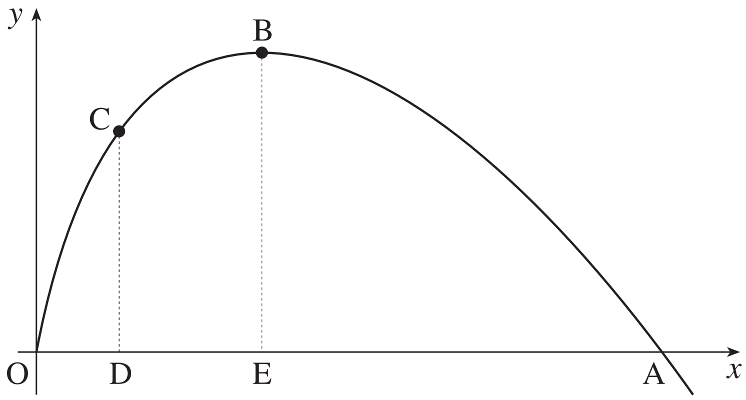
1 Fig. 7 shows the curve

$$y = 2x - x\ln x, \text{ where } x > 0.$$

The curve crosses the x-axis at A, and has a turning point at B. The point C on the curve has x-coordinate 1. Lines CD and BE are drawn parallel to the y-axis.



Not to scale

Fig. 7

(i) Find the x-coordinate of A, giving your answer in terms of e. [2]

(ii) Find the exact coordinates of B. [6]

(iii) Show that the tangents at A and C are perpendicular to each other. [3]

(iv) Using integration by parts, show that

$$\int x \ln x \, dx = \tfrac{1}{2}x^2 \ln x - \tfrac{1}{4}x^2 + c.$$

Hence find the exact area of the region enclosed by the curve, the x-axis and the lines CD and BE. [7]

2 Show that $\displaystyle\int_0^{\frac{1}{6}\pi} x \sin 2x \, dx = \frac{3\sqrt{3} \quad \pi}{24}.$ [6]

3 Fig. 8 shows part of the curve $y = x \sin 3x$. It crosses the x-axis at P. The point on the curve with x-coordinate $\frac{1}{6}\pi$ is Q.

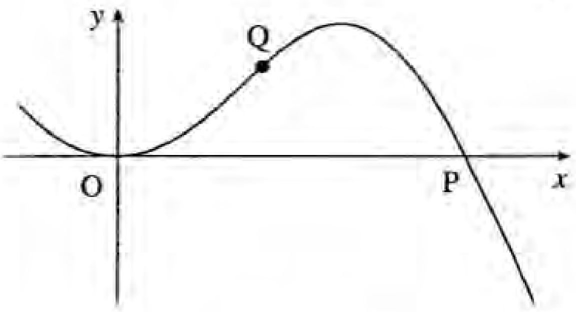


Fig. 8

(i) Find the x-coordinate of P. [3]

(ii) Show that Q lies on the line $y = x$. [1]

(iii) Differentiate $x \sin 3x$. Hence prove that the line $y = x$ touches the curve at Q. [6]

(iv) Show that the area of the region bounded by the curve and the line $y = x$ is $\frac{1}{72}(\pi^2 - 8)$. [7]

4 **(i)** Differentiate $x \cos 2x$ with respect to x. **[3]**

(ii) Integrate $x \cos 2x$ with respect to x. **[4]**