1(i) $2x - x\ln x = 0$ $\Rightarrow$ $x(2 - \ln x) = 0$ $\Rightarrow$ $(x = 0)$ or $\ln x = 2$ $\Rightarrow$ at A, $x = e^2$	M1 A1 [2]	Equating to zero
(ii) $\frac{dy}{dx} = 2 - x.\frac{1}{x} - \ln x.1$ $= 1 - \ln x$ $\frac{dy}{dx} = 0 \Rightarrow 1 - \ln x = 0$ $\Rightarrow$ $\ln x = 1$, $x = e$ When $x = e$, $y = 2e - e\ln e = e$ So B is (e, e)	M1 B1 A1 M1 A1cao B1ft [6]	Product rule for $x\ln x$ $d/dx(\ln x) = 1/x$ $1 - \ln x$ o.e. equating their derivative to zero $x = e$ $y = e$
(iii) At A, $\frac{dy}{dx} = 1 - \ln e^2 = 1 - 2$ $= -1$ At C, $\frac{dy}{dx} = 1 - \ln 1 = 1$ $1 \times -1 = -1 \Rightarrow$ tangents are perpendicular	M1 A1cao E1 [3]	Substituting x=1 or their e^2 into their derivative -1 and 1 www
(iv) Let $u = \ln x$, $dv/dx = x$ $\Rightarrow v = ½x^2$ $\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2.\frac{1}{x}dx$ $= \frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx$ $= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c$ * $A = \int_1^e (2x - x\ln x)dx$ $= \left[x^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2\right]_1^e$ $= (e^2 - ½e^2\ln e + ¼e^2) - (1 - ½1^2\ln 1 + ¼1^2)$ $= \frac{3}{4}e^2 - \frac{5}{4}$	M1 A1 E1 B1 B1 M1 A1 cao [7]	Parts: $u = \ln x$, $dv/dx = x \Rightarrow v = ½x^2$ correct integral and limits $\left[x^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2\right]$ o. substituting limits correctly

2 let $u = x$, $dv/dx = \sin 2x \Rightarrow v = -½\cos 2x$ $\Rightarrow \int_0^{\pi/6} x \sin 2x dx = \left[x.-\frac{1}{2}\cos 2x \right]_0^{\pi/6} + \int_0^{\pi/6} \frac{1}{2}\cos 2x.1.dx$ $= \frac{\pi}{6}.-\frac{1}{2}\cos\frac{\pi}{3} - 0 + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$ $= -\frac{\pi}{24} + \frac{\sqrt{3}}{8}$ $= \frac{3\sqrt{3}-\pi}{24}$ *	M1 A1 B1ft M1 B1 E1 [6]	parts with $u = x$, $dv/dx = \sin 2x$ $...+\left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$ substituting limits cos π/3 = ½ , sin π/3 = √3/2 soi www

3 (i) At P, $x \sin 3x = 0$ $\Rightarrow \sin 3x = 0$ $\Rightarrow 3x = \pi$ $\Rightarrow x = \pi/3$	M1 A1 A1cao [3]	$x \sin 3x = 0$ $3x = \pi$ or 180 $x = \pi/3$ or 1.05 or better
(ii) When $x = \pi/6$, $x \sin 3x = \frac{\pi}{6}\sin\frac{\pi}{2} = \frac{\pi}{6}$ $\Rightarrow$ Q($\pi/6$, $\pi/6$) lies on line $y = x$	E1 [1]	$y = \frac{\pi}{6}$ or $x \sin 3x = x \Rightarrow \sin 3x = 1$ etc. Must conclude in radians, and be exact
(iii) $y = x\sin 3x$ $\Rightarrow \frac{dy}{dx} = x.3\cos 3x + \sin 3x$ At Q, $\frac{dy}{dx} = \frac{\pi}{6}.3\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1$ = gradient of $y = x$ So line touches curve at this point	B1 M1 A1cao M1 A1ft E1 [6]	d/dx ($\sin 3x$) = $3\cos 3x$ Product rule consistent with their derivs $3x \cos 3x + \sin 3x$ substituting $x = \pi/6$ into their derivative = 1 ft dep 1st M1 = gradient of $y = x$ (www)
(iv) Area under curve = $\int_0^{\frac{\pi}{6}} x\sin 3x\,dx$ Integrating by parts, $u = x$, dv/dx = $\sin 3x$ $\Rightarrow v = -\frac{1}{3}\cos 3x$ $\int_0^{\frac{\pi}{6}} x\sin 3x\,dx = \left[-\frac{1}{3}x\cos 3x\right]_0^{\frac{\pi}{6}} + \int_0^{\frac{\pi}{6}} \frac{1}{3}\cos 3x\,dx$ $= -\frac{1}{3}.\frac{\pi}{6}\cos\frac{\pi}{2} + \frac{1}{3}.0.\cos 0 + \left[\frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{6}}$ $= \frac{1}{9}$ Area under line = $\frac{1}{2}\times\frac{\pi}{6}\times\frac{\pi}{6} = \frac{\pi^2}{72}$ So area required = $\frac{\pi^2}{72} - \frac{1}{9}$ $= \frac{\pi^2 - 8}{72}$ *	 M1 A1cao A1ft M1 A1 B1 E1 [7]	 Parts with $u = x$ dv/dx = $\sin 3x \Rightarrow$ $v = -\frac{1}{3}\cos 3x$ [condone no negative] $\ldots + \left[\frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{6}}$ substituting (correct) limits $\frac{1}{9}$ www $\frac{\pi^2}{72}$ www

4(i) $y = x\cos 2x$ $\Rightarrow \frac{dy}{dx} = -2x\sin 2x + \cos 2x$	M1 B1 A1 [3]	product rule d/d*x* (cos2*x*) = –2sin2*x* oe cao
(ii) $\int x\cos 2x dx = \int x \frac{d}{dx}(\frac{1}{2}\sin 2x) dx$ $= \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x dx$ $= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	M1 A1 A1ft A1 [4]	parts with $u = x$, $v = ½ \sin 2x$ $+\frac{1}{4}\cos 2x$ cao – must have + *c*