1 Find the exact value of $\int_0^2 \sqrt{1+4x}\,dx$, showing your working. **[5]**

2 Fig. 8 shows the line $y = x$ and parts of the curves $y = f(x)$ and $y = g(x)$, where

$$f(x) = e^{x-1}, \quad g(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line $y = x$ meet at the point C.

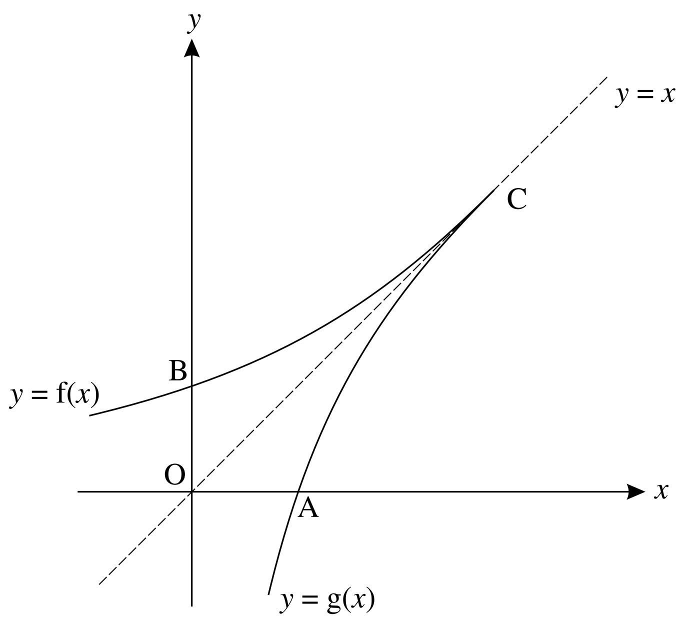

Fig. 8

(i) Find the exact coordinates of A and B. Verify that the coordinates of C are (1, 1). **[5]**

(ii) Prove algebraically that $g(x)$ is the inverse of $f(x)$. **[2]**

(iii) Evaluate $\int_0^1 f(x)\,dx$, giving your answer in terms of e. **[3]**

(iv) Use integration by parts to find $\int \ln x\,dx$.

Hence show that $\int_{e^{-1}}^1 g(x)\,dx = \frac{1}{e}$. **[6]**

(v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. **[2]**

3 A curve is defined by the equation $y = 2x \ln(1 + x)$.

(i) Find $\frac{dy}{dx}$ and hence verify that the origin is a stationary point of the curve. [4]

(ii) Find $\frac{d^2y}{dx^2}$, and use this to verify that the origin is a minimum point. [5]

(iii) Using the substitution $u = 1 + x$, show that $\int \frac{x^2}{1+x}\,dx = \int \left(u - 2 + \frac{1}{u}\right) du$.

Hence evaluate $\int_0^1 \frac{x^2}{1+x}\,dx$, giving your answer in an exact form. [6]

(iv) Using integration by parts and your answer to part **(iii)**, evaluate $\int_0^1 2x \ln(1 + x)\,dx$. [4]

4 Find $\int x e^{3x}\,dx$. [4]

5 Show that $\int_1^4 \frac{x}{x^2 + 2}\,dx = \frac{1}{2}\ln 6$. [4]