1 $\int_0^2 \sqrt{1+4x}\,dx$ let $u = 1+4x,\ du = 4dx$	M1	$u = 1 + 4x$ and $du/dx = 4$ or $du = 4dx$
$= \int_1^9 u^{1/2}.\frac{1}{4}\,du$	A1	$\int u^{1/2}.\frac{1}{4}\,du$
$= \left[\frac{1}{6}u^{3/2}\right]_1^9$	B1	$\int u^{1/2}du = \frac{u^{3/2}}{3/2}$ soi
	M1	substituting correct limits (u or x) dep attempt to integrate
$= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$ or $4\frac{1}{3}$	A1cao	
or $\frac{d}{dx}(1+4x)^{3/2} = 4.\frac{3}{2}(1+4x)^{1/2} = 6(1+4x)^{1/2}$	M1 A1	$k(1 + 4x)^{3/2}$ $\int(1+4x)^{1/2}dx = \frac{2}{3}(1+4x)^{3/2}...$
$\Rightarrow \int_0^2 (1+4x)^{1/2}dx = \left[\frac{1}{6}(1+4x)^{3/2}\right]_0^2$	A1	× ¼
$= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$ or $4\frac{1}{3}$	M1	substituting limits (dep attempt to integrate)
	A1cao [5]	

2(i) A: $1 + \ln x = 0$ $\Rightarrow$ $\ln x = -1$ so A is $(e^{-1}, 0)$ $\Rightarrow$ $x = e^{-1}$ B: $x = 0$, $y = e^{0-1} = e^{-1}$ so B is $(0, e^{-1})$ C: $f(1) = e^{1-1} = e^0 = 1$ $g(1) = 1 + \ln 1 = 1$	M1 A1 B1 E1 E1 [5]	 SC1 if obtained using symmetry condone use of symmetry Penalise A = e^{-1}, B = e^{-1}, or co-ords wrong way round, but condone labelling errors.
(ii) *Either* by invertion: e.g. $y = e^{x-1}$ $x \leftrightarrow y$ $x = e^{y-1}$ $\Rightarrow$ $\ln x = y - 1$ $\Rightarrow$ $1 + \ln x = y$	 M1 E1	 taking lns or exps
or by composing e.g. $f\,g(x) = f(1 + \ln x)$ $= e^{1+\ln x-1}$ $= e^{\ln x} = x$	 M1 E1 [2]	 $e^{1+\ln x-1}$ or $1 + \ln(e^{x-1})$
(iii) $\int_0^1 e^{x-1}dx = \left[e^{x-1}\right]_0^1$ $= e^0 - e^{-1}$ $= 1 - e^{-1}$	M1 M1 A1cao [3]	$\left[e^{x-1}\right]$ o.e or $u = x - 1 \Rightarrow \left[e^u\right]$ substituting correct limits for *x* or *u* o.e. not e^0, must be exact.
(iv) $\int \ln x\,dx = \int \ln x \frac{d}{dx}(x)dx$ $= x\ln x - \int x.\frac{1}{x}dx$ $= x \ln x - x + c$ $\Rightarrow$ $\int_{e^{-1}}^1 g(x)dx = \int_{e^{-1}}^1 (1 + \ln x)dx$ $= \left[x + x\ln x - x\right]_{e^{-1}}^1$ $= \left[x\ln x\right]_{e^{-1}}^1$ $= 1\ln 1 - e^{-1}\ln(e^{-1})$ $= e^{-1}$ *	M1 A1 A1cao B1ft DM1 E1 [6]	parts: $u = \ln x$, $du/dx = 1/x$, $v = x$, $dv/dx = 1$ condone no '*c*' ft their '$x \ln x - x$' (provided 'algebraic') substituting limits dep B1 www
(v) Area = $\int_0^1 f(x)\,d\,x - \int_{e^{-1}}^1 g(x)\,d\,x$ $= (1 - e^{-1}) - e^{-1}$ $= 1 - 2/e$	M1 A1cao	Must have correct limits 0.264 or better.
or Area OCB = area under curve – triangle $= 1 - e^{-1} - ½ \times 1 \times 1$ $= ½ - e^{-1}$ *or* Area OAC = triangle – area under curve $= ½ \times 1 \times 1 - e^{-1}$ $= ½ - e^{-1}$ Total area = $2(½ - e^{-1}) = 1 - 2/e$	 M1 A1cao [2]	 OCA or OCB = $½ - e^{-1}$ 0.264 or better

3(i) $y = 2x \ln(1 + x)$ $\Rightarrow \quad \frac{dy}{dx} = \frac{2x}{1+x} + 2\ln(1+x)$ When $x = 0$, $dy/dx = 0 + 2 \ln 1 = 0$ $\Rightarrow$ origin is a stationary point.	M1 B1 A1 E1 [4]	product rule $d/dx(\ln(1+x)) = 1/(1+x)$ soi www (i.e. from correct derivative)
(ii) $\frac{d^2y}{dx^2} = \frac{(1+x).2 - 2x.1}{(1+x)^2} + \frac{2}{1+x}$ $= \frac{2}{(1+x)^2} + \frac{2}{1+x}$ When $x = 0$, $d^2y/dx^2 = 2 + 2 = 4 > 0$ $\Rightarrow$ (0, 0) is a min point	M1 A1ft A1 M1 E1 [5]	Quotient or product rule on their $2x/(1 + x)$ correctly applied to their $2x/(1+x)$ o.e., e.g. $\frac{4+2x}{(1+x)^2}$ cao substituting $x = 0$ into their d^2y/dx^2 www – dep previous A1
(iii) Let $u = 1 + x \Rightarrow du = dx$ $\Rightarrow \int \frac{x^2}{1+x}dx = \int \frac{(u-1)^2}{u}du$ $= \int \frac{(u^2 - 2u + 1)}{u}du$ $= \int (u - 2 + \frac{1}{u})du$ * $\Rightarrow \int_0^1 \frac{x^2}{1+x}dx = \int_1^2 (u - 2 + \frac{1}{u})du$ $= \left[\frac{1}{2}u^2 - 2u + \ln u\right]_1^2$ $= 2 - 4 + \ln 2 - (½ - 2 + \ln 1)$ $= \ln 2 - ½$	M1 E1 B1 B1 M1 A1 [6]	$\frac{(u-1)^2}{u}$ www (but condone du omitted except in final answer) changing limits (or substituting back for x and using 0 and 1) $\left[\frac{1}{2}u^2 - 2u + \ln u\right]$ substituting limits (consistent with u or x) cao
(iv) $A = \int_0^1 2x\ln(1+x)dx$ Parts: $u = \ln(1 + x)$, $du/dx = 1/(1 + x)$ $dv/dx = 2x \Rightarrow v = x^2$ $= \left[x^2 \ln(1+x)\right]_0^1 - \int_0^1 \frac{x^2}{1+x}dx$ $= \ln 2 - \ln 2 + ½ = ½$	 M1 A1 M1 A1 [4]	 soi substituting their $\ln 2 - ½$ for $\int_0^1 \frac{x^2}{1+x}dx$ cao

4 Let $u = x$, $dv/dx = e^{3x} \Rightarrow v = e^{3x}/3$ $\Rightarrow \int x e^{3x}\,dx = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}.1\,dx$ $= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$	M1 A1 A1 B1 [4]	parts with $u = x$, $dv/dx = e^{3x} \Rightarrow v$ $= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}$ $+c$

5 Let $u = x^2 + 2 \Rightarrow du = 2x\,dx$ $\int_1^4 \frac{x}{x^2+2}dx = \int_3^{18} \frac{1/2}{u}du$ $= \frac{1}{2}[\ln u]_3^{18}$ = ½ (ln18 – ln 3) = ½ ln(18/3) = ½ ln 6*	M1 A1 M1 E1 [4]	$\int \frac{1/2}{u}du$ or $k \ln (x^2 + 1)$ ½ ln u or ½ ln(x^2 + 2) substituting correct limits (u or x) must show working for ln 6