1(i) At P, $x \cos 3x = 0$ $\Rightarrow$ $\cos 3x = 0$ $\Rightarrow$ $3x = \pi/2, 3\pi/2$ $\Rightarrow$ $x = \pi/6, \pi/2$ So P is $(\pi/6, 0)$ and Q is $(\pi/2, 0)$	M1 M1 A1 A1 [4]	or verification $3x = \pi/2, (3\pi/2\ldots)$ dep both Ms condone degrees here
(ii) $\frac{dy}{dx} = -3x\sin 3x + \cos 3x$ At P, $\frac{dy}{dx} = -\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = -\frac{\pi}{2}$ At TPs $\frac{dy}{dx} = -3x\sin 3x + \cos 3x = 0$ $\Rightarrow$ $\cos 3x = 3x \sin 3x$ $\Rightarrow$ $1 = 3x \sin 3x / \cos 3x = 3x \tan 3x$ $\Rightarrow$ $x \tan 3x = 1/3$ *	M1 B1 A1 M1 A1cao M1 E1 [7]	Product rule $d/dx (\cos 3x) = -3 \sin 3x$ cao (so for $dy/dx = -3x\sin 3x$ allow B1) mark final answer substituting their $-\pi/6$ (must be rads) $-\pi/2$ $dy/dx = 0$ and $\sin 3x / \cos 3x = \tan 3x$ used www
(iii) $A = \int_0^{\pi/6} x\cos 3x dx$ Parts with $u = x$, $dv/dx = \cos 3x$ $du/dx = 1$, $v = 1/3 \sin 3x$ $\Rightarrow$ $A = \left[\frac{1}{3}x\sin 3x\right]_0^{\frac{\pi}{6}} \int_0^{\pi/6}\frac{1}{3}\sin 3x dx$ $= \left[\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x\right]_0^{\frac{\pi}{6}}$ $= \frac{\pi}{18} - \frac{1}{9}$	B1 M1 A1 A1 M1dep A1 cao [6]	Correct integral and limits (soi) – ft their P, but must be in radians can be without limits dep previous A1. substituting correct limits, dep 1st M1: ft their P provided in radians o.e. but must be exact

2(i) When $x = 1$, $y = 3 \ln 1 + \quad ^{2}$ $= 0$	E1 [1]		
(ii) $\frac{dy}{dx} = \frac{3}{x} + 1 - 2x$ $\Rightarrow$ At R, $\frac{dy}{dx} = 0 = \frac{3}{x} + 1 - 2x$ $\Rightarrow$ $3 + x - 2x^2 = 0$ $\Rightarrow$ $(3 - 2x)(1 + x) = 0$ $\Rightarrow$ $x = 1.5$, (or -1) $\Rightarrow$ $y = 3 \ln 1.5 + 1.5 - 1.5^2$ $= 0.466$ (3 s f.) $\frac{d^2y}{dx^2} = -\frac{3}{x^2} - 2$ When $x = 1.5$, d^2y/dx^2 $(= -10/3)$ $< 0 \Rightarrow$ max	M1 A1cao M1 M1 A1 M1 A1cao B1ft E1 [9]	d/dx (ln x) = $1/x$ re-arranging into a quadratic = 0 factorising or formula or completing square substituting their x ft their dy/dx on equivalent work www – don't need to calculate 10/3	SC1 for $x = 1.5$ unsupported, SC3 if verified but condone rounding errors on 0.466
(iii) Let $u = \ln x$, $du/dx = 1/x$ $dv/dx = 1$, $v = x$ $\Rightarrow$ $\int \ln x dx = x \ln x - \int x . \frac{1}{x} dx$ $= x \ln x - \int 1 . dx$ $= x \ln x - x + c$ $\Rightarrow$ $A = \int_1^{2.05} (3 \ln x + x - x^2) dx$ $= \left[3x \ln x - 3x + \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_1^{2.05}$ $= -2.5057 + 2.833..$ $= 0.33$ (2 s.f.)	M1 A1 A1 B1 B1ft M1dep A1 cao [7]	parts condone no c correct integral and limits (soi) $\left[3 \times \text{their}' x \ln x - x' + \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]$ substituting correct limits dep 1st B1	 allow correct result to be quoted (SC3)

3(i) (0, ½)	B1 [1]	allow y = ½ , but not (x =) ½ or (½ ,0) nor P = 1/2	
(ii) $\frac{dy}{dx}=\frac{(1+e^{2x})2e^{2x}-e^{2x}.2e^{2x}}{(1+e^{2x})^2}$ $=\frac{2e^{2x}}{(1+e^{2x})^2}$ When x = 0, dy/dx = $2e^0/(1+e^0)^2$ = ½	M1 A1 A1 B1ft [4]	Quotient or product rule correct expression – condone missing bracket cao – mark final answer follow through their derivative	product rule: $\frac{dy}{dx}=e^{2x}.2e^{2x}(-1)(1+e^{2x})^{-2}+2e^{2x}(1+e^{2x})^{-1}$ $-\frac{2e^{2x}}{(1+e^{2x})^2}$ from (udv – vdu)/v^2 SC1
(iii) $A=\int_0^1\frac{e^{2x}}{1+e^{2x}}dx$ $=\left[\frac{1}{2}\ln(1+e^{2x})\right]_0^1$ *or* let $u = 1 + e^{2x}$, du/dx = 2 e^{2x} $\Rightarrow$ $A=\int_2^{1+e^2}\frac{1/2}{u}du$ $=\left[\frac{1}{2}\ln u\right]_2^{1+e^2}$ $=\frac{1}{2}\ln(1+e^2)-\frac{1}{2}\ln 2$ $=\frac{1}{2}\ln\left[\frac{1+e^2}{2}\right]$*	B1 M1 A1 M1 A1 M1 E1 [5]	correct integral and limits (soi) $k\ln(1+e^{2x})$ k = ½ or $v = e^{2x}$, dv/dx = $2e^{2x}$ o.e. [½ ln u] or [½ ln (v + 1)] substituting correct limits www	condone no dx allow missing dx's or incompatible limits, but penalise missing brackets
(iv) $g(-x)=\frac{1}{2}\left[\frac{e^{-x}-e^{x}}{e^{-x}+e^{x}}\right]=-\frac{1}{2}\left[\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\right]=-g(x)$ Rotational symmetry of order 2 about O	M1 E1 B1 [3]	substituting –x for x in g(x) completion www – taking out –ve must be clear must have 'rotational' 'about O', 'order 2' (oe)	not g(–x) ≠ g(x). Condone use of f for g.
(v)(A) $g(x)+\frac{1}{2}=\frac{1}{2}.\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}+\frac{1}{2}=\frac{1}{2}.(\frac{e^{x}-e^{-x}+e^{x}+e^{-x}}{e^{x}+e^{-x}})$ $=\frac{1}{2}.(\frac{2e^{x}}{e^{x}+e^{-x}})$ $=\frac{e^{x}.e^{x}}{e^{x}(e^{x}+e^{-x})}=\frac{e^{2x}}{e^{2x}+1}=f(x)$ (B) ranslation $\begin{pmatrix}0\\1/2\end{pmatrix}$ (C) Rotational symmetry [of order 2]about P	M1 A1 E1 M1 A1 B1 [6]	combining fractions (correctly) translation in y direction up ½ unit dep 'translation' used o.e. condone omission of 180°/order 2	allow 'shift', 'move' in correct direction for M1. $\begin{pmatrix}0\\1/2\end{pmatrix}$ alone is SC1.