1 Fig. 8 shows a sketch of part of the curve $y = x \sin 2x$, where x is in radians.

The curve crosses the x-axis at the point P. The tangent to the curve at P crosses the y-axis at Q.

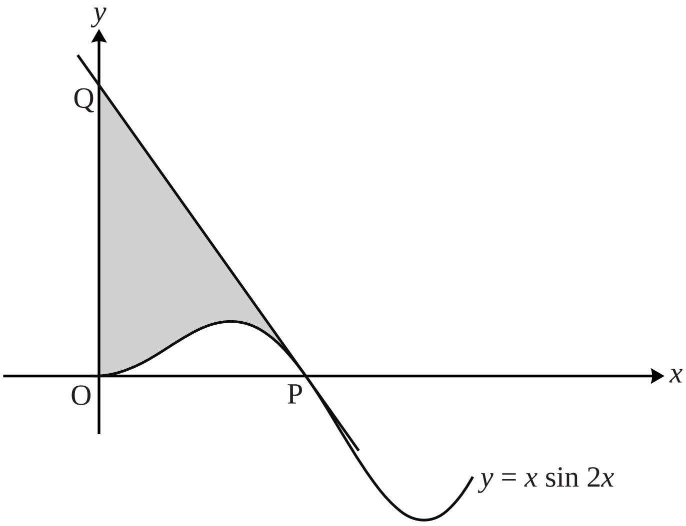

Fig. 8

(i) Find $\frac{dy}{dx}$. Hence show that the x-coordinates of the turning points of the curve satisfy the equation $\tan 2x + 2x = 0$. **[4]**

(ii) Find, in terms of π, the x-coordinate of the point P.

Show that the tangent PQ has equation $2\pi x + 2y = \pi^2$.

Find the exact coordinates of Q. **[7]**

(iii) Show that the exact value of the area shaded in Fig. 8 is $\frac{1}{8}\pi(\pi^2 - 2)$. **[7]**

2 **(i)** Use the substitution $u = 1 + x$ to show that

$$\int_0^1 \frac{x^3}{1+x}\,dx = \int_a^b \left(u^2 - 3u + 3 - \frac{1}{u}\right) du,$$

where a and b are to be found.

Hence evaluate $\int_0^1 \frac{x^3}{1+x}\,dx$, giving your answer in exact form. **[7]**

Fig. 8 shows the curve $y = x^2 \ln(1 + x)$.

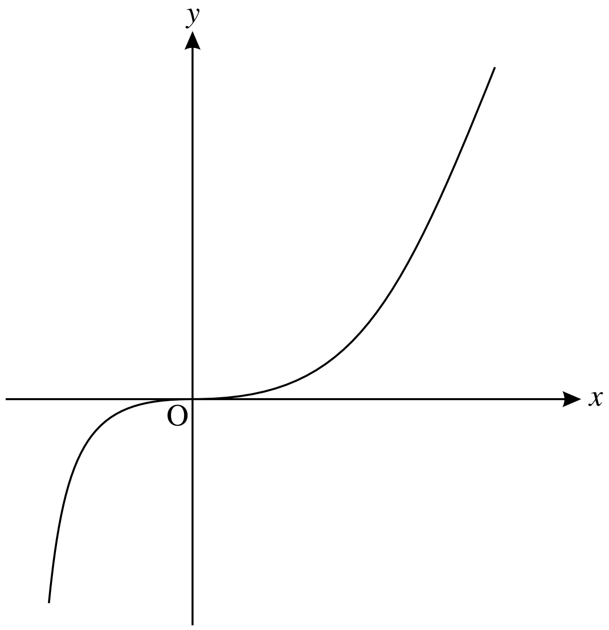

Fig. 8

(ii) Find $\frac{dy}{dx}$.

Verify that the origin is a stationary point of the curve. **[5]**

(iii) Using integration by parts, and the result of part **(i)**, find the exact area enclosed by the curve $y = x^2 \ln(1 + x)$, the x-axis and the line $x = 1$. **[6]**

3 **(i)** Differentiate $\frac{\ln x}{x^2}$, simplifying your answer. **[4]**

(ii) Using integration by parts, show that $\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x}(1 + \ln x) + c$. **[4]**

4 Evaluate the following integrals, giving your answers in exact form.

(i) $\int_0^1 \frac{2x}{x^2 + 1}\,dx$. **[3]**

(ii) $\int_0^1 \frac{2x}{x + 1}\,dx$. **[5]**