1 Find the exact value of $\int_1^2 x^3 \ln x \, dx$. **[5]**

2 Fig. 8 shows the curve $y = f(x)$, where $f(x) = \dfrac{x}{\sqrt{2+x^2}}$.

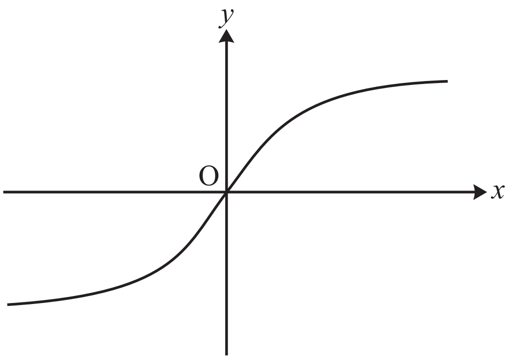

Fig. 8

(i) Show algebraically that $f(x)$ is an odd function. Interpret this result geometrically. **[3]**

(ii) Show that $f'(x) = \dfrac{2}{(2+x^2)^{\frac{3}{2}}}$. Hence find the exact gradient of the curve at the origin. **[5]**

(iii) Find the exact area of the region bounded by the curve, the x-axis and the line $x = 1$. **[4]**

(iv) (A) Show that if $y = \dfrac{x}{\sqrt{2+x^2}}$, then $\dfrac{1}{y^2} = \dfrac{2}{x^2} + 1$. **[2]**

(B) Differentiate $\dfrac{1}{y^2} = \dfrac{2}{x^2} + 1$ implicitly to show that $\dfrac{dy}{dx} = \dfrac{2y^3}{x^3}$. Explain why this expression cannot be used to find the gradient of the curve at the origin. **[4]**

3 Evaluate $\int_0^3 x(x+1)^{-\frac{1}{2}} \, dx$, giving your answer as an exact fraction. **[5]**

4 Show that $\int_0^{\frac{\pi}{2}} x\cos\tfrac{1}{2}x\,\mathrm{d}x = \frac{\sqrt{2}}{2}\pi + 2\sqrt{2} - 4.$ **[5]**

5 Fig. 8 shows the curve $y = \frac{x}{\sqrt{x-2}}$, together with the lines $y = x$ and $x = 11$.

The curve meets these lines at P and Q respectively. R is the point (11, 11).

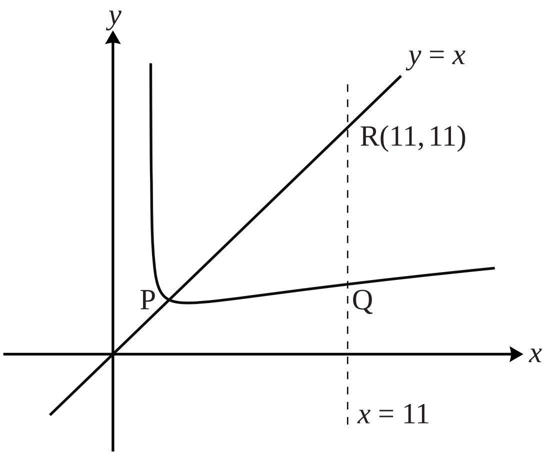

Fig. 8

(i) Verify that the x-coordinate of P is 3. **[2]**

(ii) Show that, for the curve, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x-4}{2(x-2)^{\frac{3}{2}}}$.

Hence find the gradient of the curve at P. Use the result to show that the curve is **not** symmetrical about $y = x$. **[7]**

(iii) Using the substitution $u = x - 2$, show that $\int_3^{11} \frac{x}{\sqrt{x-2}}\,\mathrm{d}x = 25\tfrac{1}{3}$.

Hence find the area of the region PQR bounded by the curve and the lines $y = x$ and $x = 11$. **[9]**