Question			Answer	Marks	Guidance	
1			let $u = \ln x$, $\mathrm{d}v/\mathrm{d}x = x^3$, $\mathrm{d}u/\mathrm{d}x = 1/x$, $v = ¼\, x^4$	M1	u, u', v', v all correct	
			$\int_1^2 x^3 \ln x \,\mathrm{d}x = \left[\frac{1}{4}x^4 \ln x\right]_1^2 - \int_1^2 \frac{1}{4}x^4 . \frac{1}{x}\mathrm{d}x$	A1	$\frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^4 . \frac{1}{x}[\mathrm{d}x]$	ignore limits
			$= \left[\frac{1}{4}x^4 \ln x\right]_1^2 \quad \int_1^2 \frac{1}{4}x^3 \mathrm{d}x$	M1dep	simplifying $x^4 / x = x^3$ in second term (soi)	dep 1st M1
			$= \left[\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4\right]_1^2$	A1cao	$\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$ o.e.	
			$= 4\ln 2 - 15/16$	A1cao	o.e. must be exact, but can isw	must evaluate ln 1 = 0 and combine $-1 + 1/16$
				[5]		

Question			Answer	Marks	Guidance	
2	(i)		$f(-x)=\frac{-x}{\sqrt{2+(-x)^2}}$	M1	substituting $-x$ for x in $f(x)$	$\frac{-x}{\sqrt{2+-x^2}},\frac{-x}{\sqrt{2+-(x^2)}},\frac{-x}{\sqrt{2+(-x^2)}}$ M1A0
			$=-\frac{x}{\sqrt{2+x^2}}=-f(x)$	A1	1st line must be shown, must have $f(-x) = -f(x)$ oe somewhere	$\frac{-x}{\sqrt{2-x^2}}$ M0A0
			Rotational symmetry of order 2 about O	B1 **[3]**	must have 'rotate' and 'O' and 'order 2 or 180 or ½ turn'	oe e.g. reflections in both x- and y-axes
	(ii)		$f'(x)=\frac{\sqrt{2+x^2}.1-x.\frac{1}{2}(2+x^2)^{-1/2}.2x}{(\sqrt{2+x^2})^2}$	M1 M1 A1	quotient or product rule used $½\,u^{-1/2}$ or $-½\,v^{-3/2}$ soi correct expression	QR: condone $u\mathrm{d}v \pm v\mathrm{d}u$, but u, v and denom must be correct $x(-1/2)(2+x^2)^{-3/2}.2x+(2+x^2)^{-1/2}$.
			$=\frac{2+x^2-x^2}{(2+x^2)^{3/2}}=\frac{2}{(2+x^2)^{3/2}}$ *	A1	**NB AG**	$=(2+x^2)^{-3/2}(-x^2+2+x^2)$
			When $x = 0$, $f'(x) = 2/2^{3/2} = 1/\sqrt{2}$	B1 **[5]**	oe e.g. $\sqrt{2}/2$, $2^{-1/2}$, $1/2^{1/2}$, but not $2/2^{3/2}$	allow isw on these seen
	(iii)		$A=\int_0^1\frac{x}{\sqrt{2+x^2}}[\mathrm{d}x]$	B1	correct integral and limits	limits may be inferred from subsequent working, condone no dx
			let $u = 2 + x^2$, $\mathrm{d}u = 2x\,\mathrm{d}x$		or $v = \sqrt{(2 + x^2)}$, $\mathrm{d}v = x(2 + x^2)^{-1/2}\mathrm{d}x$	
			$=\int_2^3\frac{1}{2}\frac{1}{\sqrt{u}}\mathrm{d}u$	M1	$\int\frac{1}{2}\frac{1}{\sqrt{u}}[\mathrm{d}u]$ or $=\int 1[\mathrm{d}v]$ or $k(2+x^2)^{1/2}$	condone no du or dv, but not $\int\frac{1}{2}\frac{1}{\sqrt{u}}\mathrm{d}x$
			$=\left[u^{1/2}\right]_2^3$	A1	$[u^{1/2}]$ o.e. (but not $1/u^{-1/2}$) or $[v]$ or $k = 1$	
			$=\sqrt{3}-\sqrt{2}$	A1cao **[4]**	must be exact	isw approximations

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2	**(iv)**	(A)	$y^2 = \frac{x^2}{2+x^2}$	M1	squaring (correctly)	must show $\left[\sqrt{(2+x^2)}\right]^2 + 2 + x^2$ (o.e.)
			$\Rightarrow \quad 1/y^2 = (2 + x^2)/x^2 = 2/x^2 + 1$ *	A1	or equivalent algebra **NB AG**	If argued backwards from given result without error, SCB1
				[2]		
	(iv)	(B)	$-2y^{-3}\mathrm{d}y/\mathrm{d}x = -4x^{-3}$	B1B1	LHS, RHS	condone $\mathrm{d}y/\mathrm{d}x\ -2y^{-3}$ unless pursued
			$\Rightarrow \quad \mathrm{d}y/\mathrm{d}x = -4x^{-3}/-2y^{-3} = 2y^3/\,x^3$ *	B1	**NB AG**	
			Not possible to substitute $x = 0$ and $y = 0$ into this expression	B1	soi (e.g. mention of 0/0)	Condone 'can't substitute $x = 0$' o.e. (i.e. need not mention $y = 0$). Condone also 'division by 0 is infinite'
				[4]		

Question			Answer	Marks	Guidance	
3			Let $u = 1 + x \Rightarrow$ $\int_0^3 x(1+x)^{-1/2}\,dx = \int_1^4 (u-1)u^{-1/2}\,du$	M1	$\int (u-1)u^{-1/2}(du)$ *	condone no du, missing bracket, ignore limits
			$= \int_1^4 (u^{1/2} - u^{-1/2})\,du$	A1	$\int (u^{1/2} - u^{-1/2})(du)$	
			$= \left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^4$	A1	$\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]$o.	e.g $\left[\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right]$; ignore limits
			= (16/3 – 4) – (2/3 – 2)	M1dep	upper–lower dep 1st M1 and integration	with correct limits e.g. 1, 4 for u or 0, 3 for x
			$= 2\frac{2}{3}$	A1cao	or $2.\dot{6}$ but must be exact	or using $w = (1+x)^{1/2} \Rightarrow$ $\int \frac{(w^2-1)2w}{w}(dw)$ M1
			OR Let $u = x$, $v' = (1 + x)^{-1/2}$	M1		$= \int 2(w^2-1)(dw)$ A1 $= \left[\frac{2}{3}w^3 - 2w\right]$ A1
			$\Rightarrow$ $u' = 1$, $v = 2(1 + x)^{1/2}$	A1		upper–lower with correct limits (w = 1,2) M1
			$\Rightarrow$ $\int_0^3 x(1+x)^{-1/2}\,dx = \left[2x(1+x)^{1/2}\right]_0^3 - \int_0^3 2(1+x)^{1/2}\,dx$	A1	ignore limits, condone no dx	8/3 A1 cao
			$= \left[2x(1+x)^{1/2} - \frac{4}{3}(1+x)^{3/2}\right]_0^3$	A1	ignor limits	*I $\int_1^4 (u-1)u^{-1/2}\,du$ done by parts:
			= (2 × 3 × 2 – 4 × 8/3) – (0 – 4/3)			$2u^{1/2}(u-1) - \int 2u^{1/2}\,du$ A1
			$= 2\frac{2}{3}$	A1cao	or $2.\dot{6}$ but must be exact	$[2u^{1/2}(u-1) - 4u^{3/2}/3]$ A1 substituting correct limits M1 8/3 A1cao
				[5]		

4			$u = x$, $du/dx = 1$, $dv/dx = \cos \frac{1}{2}x$, $v = 2\sin \frac{1}{2}x$	M1	correct u, u', v, v'	but allow v to be any multiple of $\sin \frac{1}{2}x$ M0 if $u = \cos \frac{1}{2}x$, , $v' = x$
			$\int_0^{\pi/2} x\cos\frac{1}{2}x\,dx = \left[2x\sin\frac{1}{2}x\right]_0^{\pi/2} - \int_0^{\pi/2} 2\sin\frac{1}{2}x\,dx$	A1ft	consistent with their u, v	
			$= \left[2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x\right]_0^{\pi/2}$	A1	$2x \sin \frac{1}{2}x + 4 \cos \frac{1}{2}x$ oe (no ft)	
			$= \pi\sin\frac{\pi}{4} + 4\cos\frac{\pi}{4} - (2.0.\sin 0 + 4\cos 0)$ $= \pi.\frac{1}{\sqrt{2}} + 4.\frac{1}{\sqrt{2}} - 4$	M1	substituting correct limits into correct expression	can be implied by one correct intermediate step
			$= \frac{\sqrt{2}}{2}\pi + 2\sqrt{2} - 4$*	A1cao **[5]**	**NB AG**	

5	**(i)**		When $x = 3$, $y = 3/\sqrt{(3-2)} = 3$ So P is (3, 3) which lies on $y = x$	M1 A1 **[2]**	substituting $x = 3$ (both x's) $y = 3$ and completion ('3 = 3' is enough)	or $x = x/\sqrt{(x-2)}$ M1 $\Rightarrow x = 3$ A1(by solving or verifying)
	(ii)		$\frac{dy}{dx} = \frac{\sqrt{x-2}.1 - x.\frac{1}{2}.(x-2)^{-1/2}}{x-2}$	M1 A1	Quotient or product rule PR: $-\frac{1}{2}x(x-2)^{-3/2} + (x-2)^{-1/2}$ correct expression	If correct formula stated, allow one error; otherwise QR must be on correct u and v, with numerator consistent with their derivatives and denominator correct initially
			$= \frac{x - 2 - \frac{1}{2}x}{(x-2)^{3/2}} = \frac{\frac{1}{2}x - 2}{(x-2)^{3/2}}$ $= \frac{x-4}{2(x-2)^{3/2}}$*	M1 A1	$\times$ top and bottom by $\sqrt{(x-2)}$ o.e. e.g. taking out factor of $(x-2)^{-3/2}$ NB **AG**	allow ft on correct equivalent algebra from their incorrect expression
			When $x = 3$, $dy/dx = -\frac{1}{2} \times 1^{3/2}$ $= -\frac{1}{2}$	M1 A1	substituting $x = 3$	
			This gradient would be -1 if curve were symmetrical about $y = x$	A1cao **[7]**	or an equivalent valid argument	

5	**(iii)**		$u = x - 2 \Rightarrow du/dx = 1 \Rightarrow du = dx$ When $x = 3$, $u = 1$ when $x = 11$, $u = 9$	B1	or $dx/du = 1$	No credit for integrating initial integral by parts. Condone $du = 1$.Condone missing du's in subsequent working.
			$\Rightarrow \int_3^{11} \frac{x}{\sqrt{x-2}} dx = \int_1^9 \frac{u+2}{u^{1/2}} du$	B1	$\int \frac{u+2}{u^{1/2}} (du)$	
			$= \int_1^9 (u^{1/2} + 2u^{-1/2}) du$	M1	splitting their fraction (correctly) and $u/u^{1/2} = u^{1/2}$ (or $\sqrt{u}$)	or integration by parts: $2u^{1/2}(u+2) - \int 2u^{1/2} du$ (must be fully correct – condone missing bracket
			$= \left[\frac{2}{3}u^{3/2} + 4u^{1/2}\right]_1^9$	A1	$\left[\frac{2}{3}u^{3/2} + 4u^{1/2}\right]$ (o.e)	by parts: $[2u^{1/2}(u+2) - 4u^{3/2}/3]$
			$= (18 + 12) - (2/3 + 4)$	M1	substituting correct limits	F(9) – F(1) (u) or F(11) – F(3) (x)
			$= 25\frac{1}{3}$ *	A1cao	NB AG	dep substitution and integration attempted
			Area under $y = x$ is ½ (3 + 11) × 8 = 56	B1	o.e. (e.g. 60.5 – 4.5)	
			Area = (area under $y = x$) – (area under curve)	M1	soi from working	must be trapezium area: $60.5 - 25\frac{1}{3}$ is M0
			so required area $= 56 - 25\frac{1}{3} = 30\frac{2}{3}$	A1cao **[9]**	30.7 or better	