1 Fig. 9 shows the curves $y = f(x)$ and $y = g(x)$. The function $y = f(x)$ is given by

$$f(x) = \ln\left(\frac{2x}{1+x}\right), \quad x > 0.$$

The curve $y = f(x)$ crosses the x-axis at P, and the line $x = 2$ at Q.

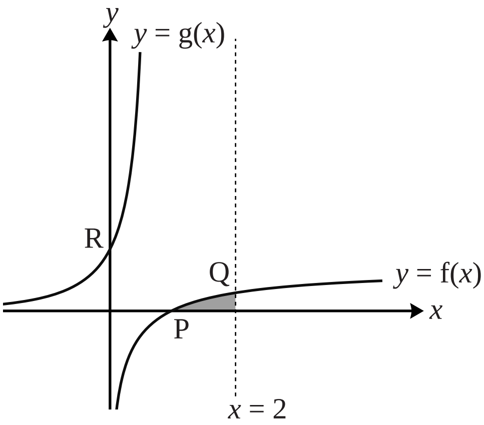

Fig. 9

(i) Verify that the x-coordinate of P is 1.

Find the exact y-coordinate of Q. **[2]**

(ii) Find the gradient of the curve at P. [Hint: use $\ln \frac{a}{b} = \ln a - \ln b$.] **[4]**

The function $g(x)$ is given by

$$g(x) = \frac{e^x}{2 - e^x}, \quad x < \ln 2.$$

The curve $y = g(x)$ crosses the y-axis at the point R.

(iii) Show that $g(x)$ is the inverse function of $f(x)$.

Write down the gradient of $y = g(x)$ at R. **[5]**

(iv) Show, using the substitution $u = 2 - e^x$ or otherwise, that $\int_0^{\ln \frac{4}{3}} g(x)\,dx = \ln \frac{3}{2}$.

Using this result, show that the exact area of the shaded region shown in Fig. 9 is $\ln \frac{32}{27}$.
[Hint: consider its reflection in $y = x$.] **[7]**

2 Fig. 8 shows the line $y = x$ and parts of the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, where

$$\mathrm{f}(x) = \mathrm{e}^{x-1}, \quad \mathrm{g}(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line $y = x$ meet at the point C.

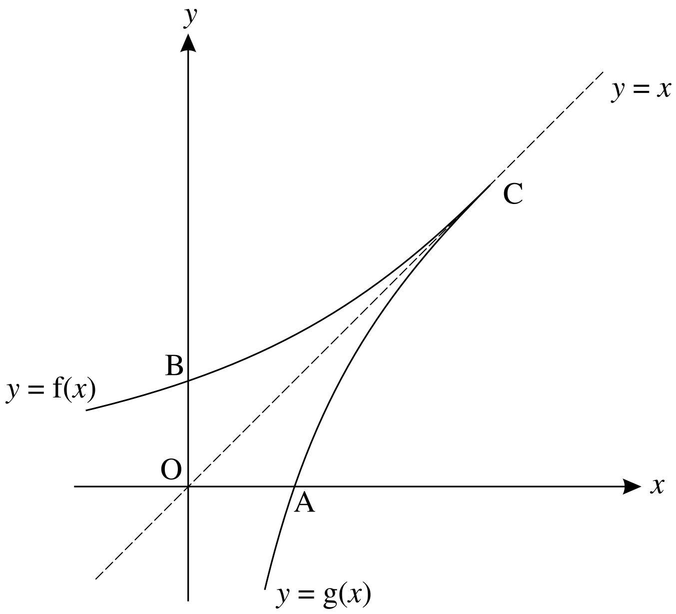

Fig. 8

(i) Find the exact coordinates of A and B. Verify that the coordinates of C are (1, 1). **[5]**

(ii) Prove algebraically that $\mathrm{g}(x)$ is the inverse of $\mathrm{f}(x)$. **[2]**

(iii) Evaluate $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in terms of e. **[3]**

(iv) Use integration by parts to find $\int \ln x \,\mathrm{d}x$.

Hence show that $\int_{\mathrm{e}^{-1}}^{1} \mathrm{g}(x)\,\mathrm{d}x = \frac{1}{\mathrm{e}}$. **[6]**

(v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. **[2]**

3 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = 1 + \sin 2x$ for $-\frac{1}{4}\pi \leqslant x \leqslant \frac{1}{4}\pi$.

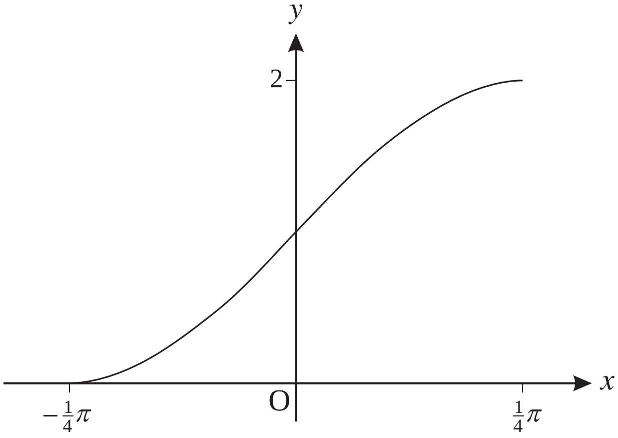

Fig. 8

(i) State a sequence of two transformations that would map part of the curve $y = \sin x$ onto the curve $y = \mathrm{f}(x)$. [4]

(ii) Find the area of the region enclosed by the curve $y = \mathrm{f}(x)$, the x-axis and the line $x = \frac{1}{4}\pi$. [4]

(iii) Find the gradient of the curve $y = \mathrm{f}(x)$ at the point $(0, 1)$. Hence write down the gradient of the curve $y = \mathrm{f}^{-1}(x)$ at the point $(1, 0)$. [4]

(iv) State the domain of $\mathrm{f}^{-1}(x)$. Add a sketch of $y = \mathrm{f}^{-1}(x)$ to a copy of Fig. 8. [3]

(v) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

4 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{1+\cos x}$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

P is the point on the curve with x-coordinate $\frac{1}{3}\pi$.

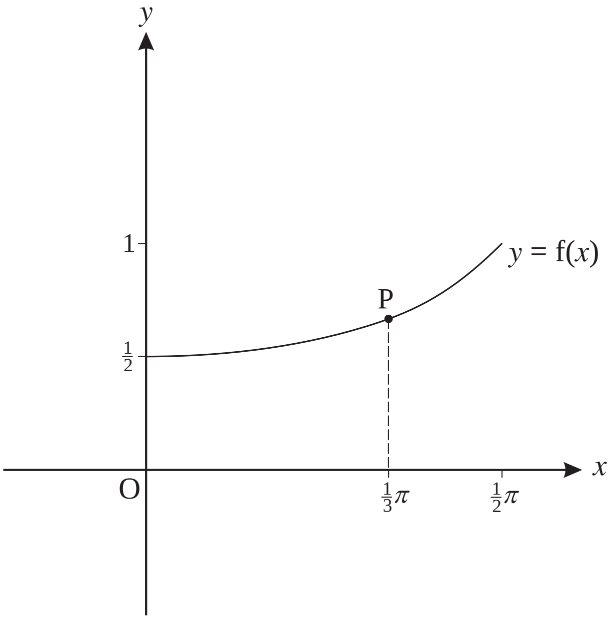

Fig. 8

(i) Find the y-coordinate of P. [1]

(ii) Find $\mathrm{f}'(x)$. Hence find the gradient of the curve at the point P. [5]

(iii) Show that the derivative of $\dfrac{\sin x}{1+\cos x}$ is $\dfrac{1}{1+\cos x}$. Hence find the exact area of the region enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \frac{1}{3}\pi$. [7]

(iv) Show that $\mathrm{f}^{-1}(x) = \arccos\left(\dfrac{1}{x} - 1\right)$. State the domain of this inverse function, and add a sketch of $y = \mathrm{f}^{-1}(x)$ to a copy of Fig. 8. [5]