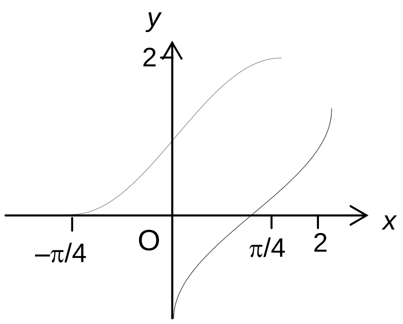
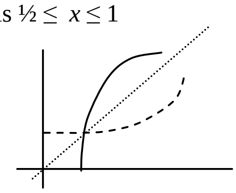
1	**(i)**		When x = 1, f(1) =ln(2/2)= ln 1 = 0 so P is (1, 0) f(2) = ln(4/3)	B1 B1 **[2]**	or ln(2x/1+x) = 0 ⇒ 2x/(1+x) = 1 ⇒ 2x = 1+x ⇒ x = 1	 if approximated, can isw after ln(4/3)
	(ii)		y = ln (2x) – ln(1 + x) $\Rightarrow \quad \frac{dy}{dx}=\frac{2}{2x}-\frac{1}{1+x}$ **OR** $\frac{d}{dx}(\frac{2x}{1+x})=\frac{(1+x)2-2x.1}{(1+x)^2}=\frac{2}{(1+x)^2}$ $\frac{dy}{dx}=\frac{2}{(1+x)^2}.\frac{1}{2x/(1+x)} \quad =\frac{1}{x(1+x)}$ At P, dy/dx = 1 – ½ = ½	M1 M1 A1cao B1 M1 A1 A1cao **[4]**	 one term correct mark final ans correct quotient or product rule chain rule attempted o.e., but mark final ans	condone lack of brackets 2/2x or −1/(1+x) need not be simplified need not be simplified

1	**(iii)**		$x = \ln[2y/(1+y)]$ or $\Rightarrow e^x = 2y/(1+y)$ $\Rightarrow e^x(1+y) = 2y$ $\Rightarrow e^x = 2y - e^x y = y(2 - e^x)$ $\Rightarrow y = e^x/(2 - e^x)$ [= g(x)] **OR** $gf(x)=g(2x/(1+x)) = e^{\ln[2x/(1+x)]}/\{2-e^{\ln[2x/(1+x)]}\}$ $= \dfrac{2x/(1+x)}{2-2x/(1+x)}$ $= \dfrac{2x}{2+2x-2x} = \dfrac{2x}{2} = x$ gradient at R = 1/ ½ = 2	 B1 B1 B1 B1 M1 A1 M1A1 B1 ft **[5]**	($x \leftrightarrow y$ here or at end to complete) completion forming gf or fg 1/their ans in (ii) unless ±1 or 0	$x = e^y/(2 - e^y)$ $x(2 - e^y) = e^y$ B1 $2x = e^y + xe^y = e^y(1 + x)$ B1 $2x/(1+x) = e^y$ B1 $\ln[2x/(1+x)] = y$ [= f(x)] B1 $fg(x) = \ln\{2e^x/(2-e^x)/[1+ e^x/(2-e^x)]\}$ M1 $= \ln[2e^x/(2 - e^x + e^x)]$ A1 $= \ln(e^x) = x$ M1A1 2 must follow ½ for 9(ii) unless g′(x) used (*see additional notes*)
	(iv)		let $u = 2 - e^x \Rightarrow du/dx = -e^x$ $x = 0$, $u = 1$, $x = \ln(4/3)$, $u = 2 - 4/3 = 2/3$ $\Rightarrow \int_0^{\ln(4/3)} g(x)\,dx = \int_1^{2/3} -\frac{1}{u}\,du$ $= \left[-\ln(u)\right]_1^{2/3} = -\ln(2/3) + \ln 1 = \ln(3/2)$* Shaded region = rectangle – integral = 2ln(4/3) – ln(3/2) = ln(16/9 × 2/3) = ln(32/27)*	 B1 M1 A1 A1cao M1 B1 A1cao **[7]**	 $2-e^0 = 1$, and $2 - e^{\ln(4/3)} = 2/3$ seen $\int -1/u\,du$ condone $\int 1/u\,du$ [–ln(u)] (could be [ln$u$] if limits swapped) NB AG rectangle area = 2ln(4/3) NB AG must show at least one step from 2ln(4/3) – ln(3/2)	 here or later (i.e. after substituting 0 and ln(4/3) into $\ln(2 - e^x)$) or by inspection [$k \ln(2 - e^x)$] $k = -1$ Allow full marks here for correctly evaluating $\int_1^2 \ln(\frac{2x}{1+x})\,dx$ (*see additional notes)*

2(i) A: $1 + \ln x = 0$ $\Rightarrow$ $\ln x = -1$ so A is $(e^{-1}, 0)$ $\Rightarrow$ $x = e^{-1}$ B: $x = 0$, $y = e^{0-1} = e^{-1}$ so B is $(0, e^{-1})$ C: $f(1) = e^{1-1} = e^0 = 1$ $g(1) = 1 + \ln 1 = 1$	M1 A1 B1 E1 E1 [5]	 SC1 if obtained using symmetry condone use of symmetry Penalise A = e^{-1}, B = e^{-1}, or co-ords wrong way round, but condone labelling errors.
(ii) *Either* by invertion: e.g. $y = e^{x-1}$ $x \leftrightarrow y$ $x = e^{y-1}$ $\Rightarrow$ $\ln x = y - 1$ $\Rightarrow$ $1 + \ln x = y$	 M1 E1	 taking lns or exps
or by composing e.g. $f\,g(x) = f(1 + \ln x)$ $= e^{1 + \ln x - 1}$ $= e^{\ln x} = x$	 M1 E1 [2]	 $e^{1 + \ln x - 1}$ or $1 + \ln(e^{x-1})$
(iii) $\int_0^1 e^{x-1}dx = \left[e^{x-1}\right]_0^1$ $= e^0 - e^{-1}$ $= 1 - e^{-1}$	M1 M1 A1cao [3]	$\left[e^{x-1}\right]$ o.e or $u = x - 1 \Rightarrow \left[e^u\right]$ substituting correct limits for x or u o.e. not e^0, must be exact.
(iv) $\int \ln x dx = \int \ln x \frac{d}{dx}(x)dx$ $= x\ln x - \int x.\frac{1}{x}dx$ $= x \ln x - x + c$ $\Rightarrow$ $\int_{e^{-1}}^1 g(x)dx = \int_{e^{-1}}^1 (1+\ln x)dx$ $= \left[x + x\ln x - x\right]_{e^{-1}}^1$ $= \left[x\ln x)\right]_{e^{-1}}^1$ $= 1\ln 1 - e^{-1}\ln(e^{-1})$ $= e^{-1}$ *	M1 A1 A1cao B1ft DM1 E1 [6]	parts: $u = \ln x$, $du/dx = 1/x$, $v = x$, $dv/dx = 1$ condone no 'c' ft their '$x \ln x - x$' (provided 'algebraic') substituting limits dep B1 www
(v) Area = $\int_0^1 f(x)\,d\,x - \int_{e^{-1}}^1 g(x)\,d\,x$ $= (1 - e^{-1}) - e^{-1}$ $= 1 - 2/e$	M1 A1cao	Must have correct limits 0.264 or better.
or Area OCB = area under curve – triangle $= 1 - e^{-1} - ½ \times 1 \times 1$ $= ½ - e^{-1}$ *or* Area OAC = triangle – area under curve $= ½ \times 1 \times 1 - e^{-1}$ $= ½ - e^{-1}$ Total area = $2(½ - e^{-1}) = 1 - 2/e$	 M1 A1cao [2]	 OCA or OCB = $½ - e^{-1}$ 0.264 or better

3 (i) Stretch in x-direction s.f. translation in y-direction 1 unit up	M1 A1 M1 A1 [4]	(in either order) – allow 'contraction' dep 'stretch' allow 'move', 'shift', etc – direction can be inferred from $\begin{pmatrix}0\\1\end{pmatrix}$ or $\begin{pmatrix}0\\1\end{pmatrix}$ dep 'translation'. $\begin{pmatrix}0\\1\end{pmatrix}$ alone is M1 A0
(ii) $A=\int_{-\pi/4}^{\pi/4}(1+\sin 2x)dx$ $=\left[x-\frac{1}{2}\cos 2x\right]_{-\pi/4}^{\pi/4}$ $= \pi/4 - ½ \cos \pi/2 + \pi/4 + ½ \cos (-\pi/2)$ $= \pi/2$	M1 B1 M1 A1 [4]	correct integral and limits. Condone dx missing; limits may be implied from subsequent working. substituting their limits (if zero lower limit used, must show evidence of substitution) or 1.57 or better – cao (www)
(iii) $y = 1 + \sin 2x$ ⇒ dy/dx = 2cos 2x When x = 0, dy/dx = 2 So gradient at (0, 1) on f(x) is 2 ⇒ gradient at (1, 0) on $f^{-1}(x)$ = ½	M1 A1 A1ft B1ft [4]	differentiating – allow 1 error (but not x + 2cos 2x) If 1, then must show evidence of using reciprocal, e.g. 1/1
(iv) Domain is $0 \le x \le 2$. 	B1 M1 A1 [3]	Allow 0 to 2, but not $0 < x < 2$ or y instead of x clear attempt to reflect in $y = x$ correct domain indicated (0 to 2), and reasonable shape
(v) $y = 1 + \sin 2x$ $x \leftrightarrow y$ $x = 1 + \sin 2y$ ⇒ $\sin 2y = x - 1$ ⇒ $2y = \arcsin(x - 1)$ ⇒ $y = ½ \arcsin(x - 1)$	M1 A1 [2]	or $\sin 2x = y - 1$ cao

4(i) $y = 1/(1+\cos\pi/3) = 2/3$.	B1 [1]	or 0.67 or better
(ii) $f'(x) = -1(1+\cos x)^{-2}.-\sin x$ $= \frac{\sin x}{(1+\cos x)^2}$ When $x = \pi/3$, $f'(x) = \frac{\sin(\pi/3)}{(1+\cos(\pi/3))^2}$ $= \frac{\sqrt{3}/2}{(1\frac{1}{2})^2} = \frac{\sqrt{3}}{2}\times\frac{4}{9} = \frac{2\sqrt{3}}{9}$	M1 B1 A1 M1 A1 [5]	chain rule or quotient rule $d/dx\ (\cos x) = -\sin x$ soi correct expression substituting $x = \pi/3$ oe or 0.38 or better. (0.385, 0.3849)
(iii) deriv = $\frac{(1+\cos x)\cos x - \sin x.(-\sin x)}{(1+\cos x)^2}$ $= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ $= \frac{\cos x + 1}{(1+\cos x)^2}$ $= \frac{1}{1+\cos x}$ * Area = $\int_0^{\pi/3} \frac{1}{1+\cos x}\,dx$ $= \left[\frac{\sin x}{1+\cos x}\right]_0^{\pi/3}$ $= \frac{\sin\pi/3}{1+\cos\pi/3}(-0)$ $= \frac{\sqrt{3}}{2}\times\frac{2}{3} = \frac{\sqrt{3}}{3}$	M1 A1 M1dep E1 B1 M1 A1 cao [7]	Quotient or product rule – condone uv' – u'v for M1 correct expression $\cos^2 x + \sin^2 x = 1$ used dep M1 www substituting limits or $1/\sqrt{3}$ - must be exact
(iv) $y = 1/(1 + \cos x)$ $x \leftrightarrow y$ $x = 1/(1 + \cos y)$ $\Rightarrow$ $1 + \cos y = 1/x$ $\Rightarrow$ $\cos y = 1/x - 1$ $\Rightarrow$ $y = \arccos(1/x - 1)$ * Domain is $½ \le x \le 1$ 	M1 A1 E1 B1 B1 [5]	attempt to invert equation www reasonable reflection in $y = x$