1 Given that $f(x) = \dfrac{x+1}{x-1}$, show that $ff(x) = x$.

Hence write down the inverse function $f^{-1}(x)$. What can you deduce about the symmetry of the curve $y = f(x)$? **[5]**

2 The functions $f(x)$ and $g(x)$ are defined for all real numbers x by

$$f(x) = x^2, \qquad g(x) = x - 2.$$

(i) Find the composite functions $fg(x)$ and $gf(x)$. [3]

(ii) Sketch the curves $y = f(x)$, $y = fg(x)$ and $y = gf(x)$, indicating clearly which is which. [2]

3 Given that $f(x) = 1 - x$ and $g(x) = |x|$, write down the composite function $gf(x)$.

On separate diagrams, sketch the graphs of $y = f(x)$ and $y = gf(x)$. [3]

4 The function $f(x)$ is defined by $f(x) = 1 + 2\sin x$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Show that $f^{-1}(x) = \arcsin\left(\frac{x \quad 1}{2}\right)$ and state the domain of this function. [4]

Fig. 6 shows a sketch of the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

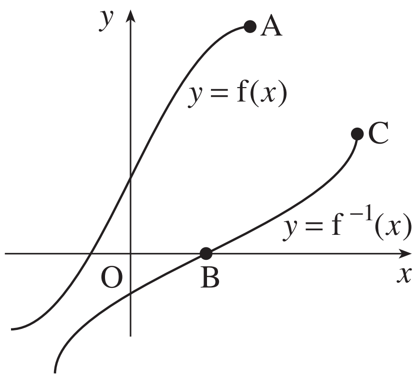

Fig. 6

(ii) Write down the coordinates of the points A, B and C. [3]

5 Given that $\arcsin x = \frac{1}{6}\pi$, find x. Find $\arccos x$ in terms of π. [3]

6 The functions $f(x)$ and $g(x)$ are defined for the domain $x > 0$ as follows:

$$f(x) = \ln x, \quad g(x) = x^3.$$

Express the composite function $fg(x)$ in terms of $\ln x$.

State the transformation which maps the curve $y = f(x)$ onto the curve $y = fg(x)$. [3]

7 Fig. 9 shows the line $y = x$ and the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{1}{2}(\mathrm{e}^x - 1)$. The line and the curve intersect at the origin and at the point $\mathrm{P}(a, a)$.

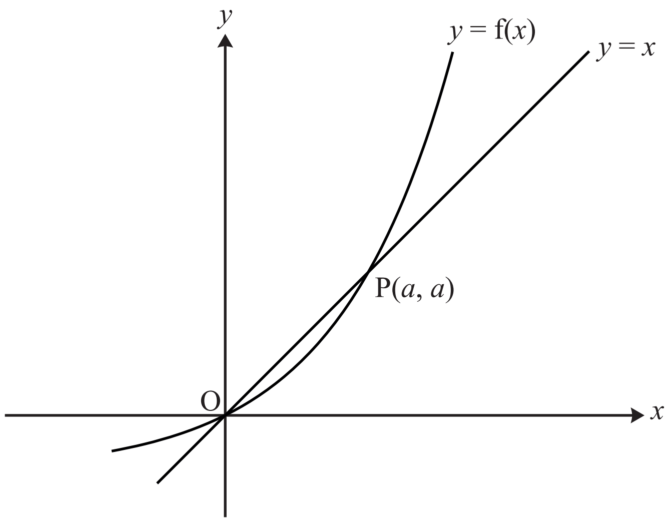

Fig. 9

(i) Show that $\mathrm{e}^a = 1 + 2a$. **[1]**

(ii) Show that the area of the region enclosed by the curve, the x-axis and the line $x = a$ is $\frac{1}{2}a$. Hence find, in terms of a, the area enclosed by the curve and the line $y = x$. **[6]**

(iii) Show that the inverse function of $\mathrm{f}(x)$ is $\mathrm{g}(x)$, where $\mathrm{g}(x) = \ln(1 + 2x)$. Add a sketch of $y = \mathrm{g}(x)$ to the copy of Fig. 9. **[5]**

(iv) Find the derivatives of $\mathrm{f}(x)$ and $\mathrm{g}(x)$. Hence verify that $\mathrm{g}'(a) = \dfrac{1}{\mathrm{f}'(a)}$.

Give a geometrical interpretation of this result. **[7]**

8 The function $f(x)$ is defined by $f(x) = 1 - 2\sin x$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$. Fig. 3 shows the curve $y = f(x)$.

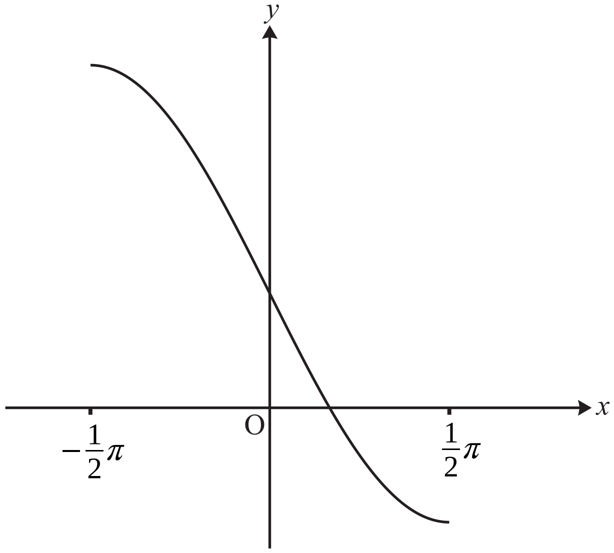

Fig. 3

(i) Write down the range of the function $f(x)$. **[2]**

(ii) Find the inverse function $f^{-1}(x)$. **[3]**

(iii) Find $f'(0)$. Hence write down the gradient of $y = f^{-1}(x)$ at the point $(1, 0)$. **[3]**