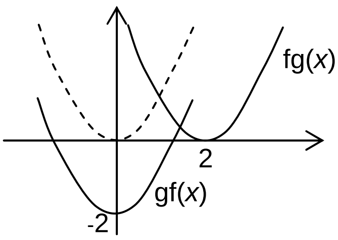
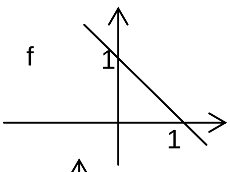
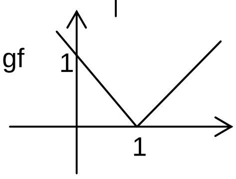
1 $\mathrm{f\,f}(x)=\dfrac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$ $=\dfrac{x+1+x-1}{x+1-x+1}$ $= 2x/2 = x$* $\mathrm{f}^{-1}(x) = \mathrm{f}(x)$ Symmetrical about $y = x$.	M1 M1 E1 B1 B1 [5]	correct expression without subsidiary denominators e.g. $=\dfrac{x+1+x-1}{x-1}\times\dfrac{x-1}{x+1-x+1}$ stated, or shown by inverting

2 (i) fg(x) = f(x – 2) = $(x – 2)^2$ gf(x) = g(x^2) = x^2 – 2.	M1 A1 A1 [3]	forming a composite function mark final answer If fg and gf the wrong way round, M1A0A0
(ii) 	B1ft B1ft [2]	fg – must have (2, 0)labelled (or inferable from scale). Condone no y-intercept, unless wrong gf – must have (0, –2) labelled (or inferable from scale) Condone no x-intercepts, unless wrong Allow ft only if fg and gf are correct but wrong way round.

3 gf(x) = $\lvert 1-x \rvert$  	B1 B1 B1 [3]	 intercepts must be labelled line must extend either side of each axis condone no labels, but line must extend to left of y axis

4(i) $y = 1 + 2\sin x \quad y \leftrightarrow x$ $\Rightarrow \quad x = 1 + 2\sin y$ $\Rightarrow \quad x - 1 = 2\sin y$ $\Rightarrow \quad (x - 1)/2 = \sin y$ $\Rightarrow \quad y = \arcsin(\frac{x-1}{2})$ * Domain is $-1 \le x \le 3$ **(ii)** A is $(\pi/2, 3)$ B is $(1, 0)$ C is $(3, \pi/2)$	M1 A1 E1 B1 B1cao B1cao B1ft [7]	Attempt to invert Allow $\pi/2 = 1.57$or better ft on their A

5 $x = ½$ $\cos \theta = ½$ $\Rightarrow \quad \theta = \pi/3$	B1 M1 A1 [3]	 M1A0 for 1.04… or 60°

6 $fg(x) = \ln(x^3)$ $= 3 \ln x$ Stretch s.f. 3 in y direction	M1 A1 B1 [3]	$\ln(x^3)$ $= 3 \ln x$

7	**(i)**		At P(a, a) g(a) = a so ½(e^a – 1) = a $\Rightarrow$ $e^a = 1 + 2a$ *	B1 **[1]**	**NB AG**	
	(ii)		$A=\int_0^a \frac{1}{2}(e^x-1)\,dx$	M1	correct integral and limits	limits can be implied from subsequent work
			$=\frac{1}{2}\left[e^x-x\right]_0^a$	B1	integral of $e^x - 1$ is $e^x - x$	
			= ½ ($e^a - a - e^0$)	A1		
			= ½ (1 + 2a – a – 1) = ½ a *	A1	**NB AG**	
			area of triangle = ½ a^2	B1		
			area between curve and line = ½ a^2 – ½ a	B1cao **[6]**	mark final answer	
	(iii)		y = ½(e^x – 1) swap x and y x = ½ (e^y – 1)			
			$\Rightarrow$ $2x = e^y - 1$	M1	Attempt to invert – one valid step	merely swapping x and y is not 'one step'
			$\Rightarrow$ $2x + 1 = e^y$	A1		
			$\Rightarrow$ ln(2x + 1) = y *	A1	y = ln(2x + 1) or g(x) = ln(2x + 1) **AG**	apply a similar scheme if they start with g(x) and invert to get f(x).
			$\Rightarrow$ g(x) = ln(2x + 1)			or g f(x) = g((e^x – 1)/2) M1
			Sketch: recognisable attempt to reflect in $y = x$	M1	through O and (a, a)	= ln(1 + e^x – 1) = ln(e^x) A1 = x A1
			Good shape	A1 **[5]**	no obvious inflexion or TP, extends to third quadrant, without gradient becoming too negative	similar scheme for fg See appendix for examples

7	**(iv)**		$f'(x) = ½e^x$ $g'(x) = 2/(2x + 1)$ $g'(a) = 2/(2a + 1)$, $f'(a) = ½e^a$ so $g'(a) = 2/e^a$ or $f'(a) = ½(2a+1)$ $= 1/(½e^a)$ $= (\ a + 1)/2$ $[= 1/f'(a)]$ $[= 1\ \ '(a)]$ tangents are reflections in $y = x$	B1 M1 A1 B1 M1 A1 B1 **[7]**	$1/(2x + 1)$ (or $1/u$ with $u = 2x + 1$) … … × 2 to get $2/(2x + 1)$ either $g'(a)$ or $f'(a)$ correct soi substituting $e^a = 1 + 2a$ establishing $f'(a) = 1/g'(a)$ must mention tangents	either way round

8	**(i)**		Range is $-1 \le y \le 3$	M1 A1 **[2]**	$-1, 3$ $-1 \le y \le 3$ or $-1 \le f(x) \le 3$ or $[-1, 3]$ (not -1 to 3, $-1 \le x \le 3$, $-1 < y < 3$ etc)
	(ii)		$y = 1 - 2\sin x$ $x \leftrightarrow y$ $x = 1 - 2\sin y \Rightarrow x - 1 = -2\sin y$ $\Rightarrow$ $\sin y = (1 - x)/2$ $\Rightarrow$ $y = \arcsin[(1 - x)/2]$	 M1 A1 A1 **[3]**	[can interchange x and y at any stage] attempt to re-arrange o.e. e.g. $\sin y = (x - 1)/(-2)$ (or $\sin x = (y - 1)/(-2)$) or $f^{-1}(x) = \arcsin[(1 - x)/2]$, not x or $f^{-1}(y) = \arcsin[1 - y)/2]$ (viz must have swapped x and y for final 'A' mark). $\arcsin[(x - 1)/-2]$ is A0
	(iii)		$f'(x) = -2\cos x$ $\Rightarrow$ $f'(0) = -2$ $\Rightarrow$ gradient of $y = f^{-1}(x)$ at $(1, 0) = -½$	M1 A1 A1 **[3]**	condone $2\cos x$ cao not $1/-2$