1 **(i)** The function $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = \frac{1-x}{1+x}, x \neq -1.$$

Show that $\mathrm{f}(\mathrm{f}(x)) = x$.

Hence write down $\mathrm{f}^{-1}(x)$. **[3]**

(ii) The function $\mathrm{g}(x)$ is defined for all real x by

$$\mathrm{g}(x) = \frac{1-x^2}{1+x^2}.$$

Prove that $\mathrm{g}(x)$ is even. Interpret this result in terms of the graph of $y = \mathrm{g}(x)$. **[3]**

2 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (\mathrm{e}^x - 2)^2 - 1, \ x \in \mathbb{R}.$$

The curve crosses the x-axis at O and P, and has a turning point at Q.

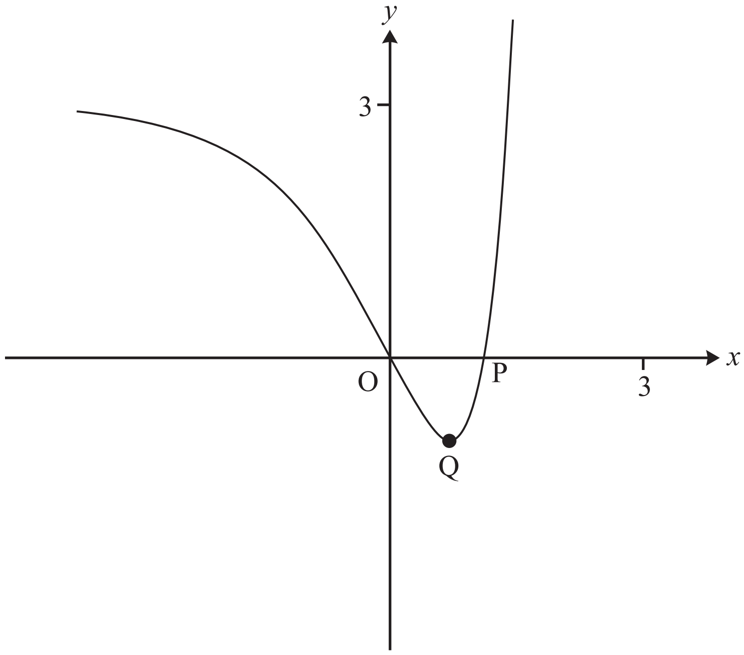

Fig. 9

(i) Find the exact x-coordinate of P. **[2]**

(ii) Show that the x-coordinate of Q is $\ln 2$ and find its y-coordinate. **[4]**

(iii) Find the exact area of the region enclosed by the curve and the x-axis. **[5]**

The domain of $\mathrm{f}(x)$ is now restricted to $x \geqslant \ln 2$.

(iv) Find the inverse function $\mathrm{f}^{-1}(x)$. Write down its domain and range, and sketch its graph on the copy of Fig. 9.

[7]

3 Fig. 7 shows the curve $y = f(x)$, where $f(x) = 1 + 2\arctan x$, $x \in \mathbb{R}$. The scales on the x- and y-axes are the same.

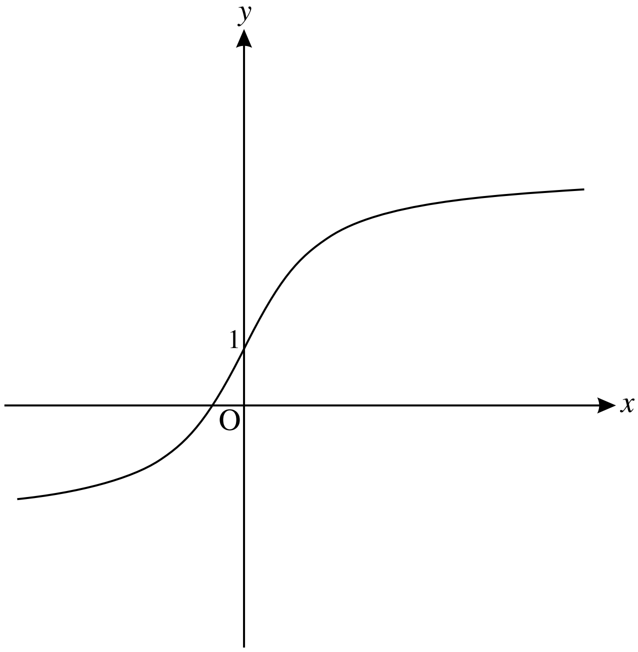

Fig. 7

(i) Find the range of f, giving your answer in terms of π. **[3]**

(ii) Find $f^{-1}(x)$, and add a sketch of the curve $y = f^{-1}(x)$ to the copy of Fig. 7. **[5]**

4 Given that $f(x) = 2\ln x$ and $g(x) = e^x$, find the composite function $gf(x)$, expressing your answer as simply as possible. **[3]**

5 Write down the conditions for $f(x)$ to be an odd function and for $g(x)$ to be an even function.

Hence prove that, if $f(x)$ is odd and $g(x)$ is even, then the composite function $gf(x)$ is even. **[4]**

6 The function $f(x)$ is defined by

$$f(x) = 1 + 2\sin 3x, \quad -\frac{\pi}{6} \leqslant x \leqslant \frac{\pi}{6}.$$

You are given that this function has an inverse, $f^{-1}(x)$.

Find $f^{-1}(x)$ and its domain. **[6]**

7 Given that $f(x) = \frac{1}{2}\ln(x-1)$ and $g(x) = 1 + e^{2x}$, show that $g(x)$ is the inverse of $f(x)$. **[3]**

8 Sketch the curve $y = 2\arccos x$ for $-1 \leqslant x \leqslant 1$. **[3]**