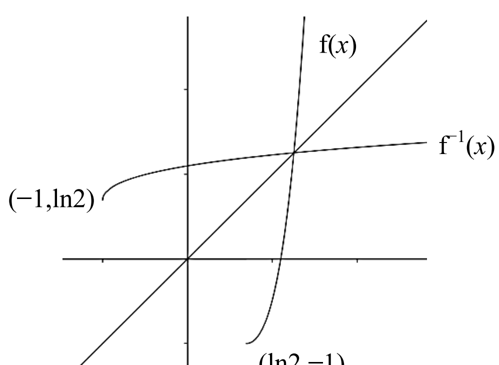
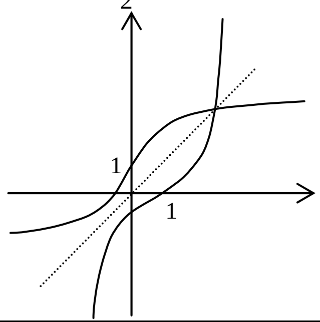
1	**(i)**		$ff(x) = f\left(\frac{1-x}{1+x}\right) = \dfrac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}$	M1	substituting $(1-x)/(1+x)$ for x in $f(x)$	
			$= \dfrac{1+x-1+x}{1+x+1-x} = \dfrac{2x}{2} = x$*	A1	correctly simplified to x **NB AG**	
			$f^{-1}(x) = f(x) = (1-x)/(1+x)$	B1	or just $f^{-1}(x) = f(x)$	
				[3]		
	(ii)		$g(-x) = \dfrac{1-(-x)^2}{1+(-x)^2}$	M1	substituting $-x$ for x in $g(x)$ condone use of 'f' for g	if brackets are omitted or misplaced allow M1A0
			$= \dfrac{1-x^2}{1+x^2} = g(x)$	A1	must indicate that $g(-x) = g(x)$ somewhere	condone use of 'f' for g
			Graph is symmetrical about the y-axis.	B1	allow 'reflected', 'reflection' for symmetrical	must state axis (y-axis or $x = 0$)
				[3]		

2	**(i)**		At P, $(e^x - 2)^2 - 1 = 0$ $\Rightarrow \quad e^x - 2 = [\pm]1$, $e^x = [1 \text{ or}]\ 3$	M1	square rooting – condone no ±	
			or $(e^x)^2 - 4e^x + 3 = 0$ $\Rightarrow (e^x - 1)(e^x - 3) = 0$, $e^x = 1$ or 3	M1	expanding to correct quadratic and solve by factorising or using quadratic formula	condone e^x^2
			$\Rightarrow x = [0 \text{ or}]\ \ln 3$	A1 **[2]**	x-coordinate of P is ln 3; must be exact	condone P = ln 3, but not y = ln 3
2	**(ii)**		$f'(x) = 2(e^x - 2)e^x$ $= 0$ when $e^x = 2$, $x = \ln 2$ *	M1 A1 A1	chain rule correct derivative not from wrong working **NB AG**	e.g. 2 u × their deriv of e^x $2(e^x - 2)x$ is M0 or verified by substitution
			or $f(x) = e^{2x} - 4e^x + 3$ $\Rightarrow f'(x) = 2e^{2x} - 4e^x$ $= 0$ when $2e^{2x} = 4e^x$, $e^x = 2$, $x = \ln 2$ *	M1 A1 A1	expanding to 3 term quadratic with $(e^x)^2$ or e^{2x} correct derivative, not from wrong working or $2e^x(e^x - 2) = 0 \Rightarrow e^x = 2$, $x = \ln 2$ not from wrong working **NB AG**	condone e^x^2 or verified by substitution
			$y = f(\ln(2)) = -1$	B1 **[4]**		

Question			Answer	Marks	Guidance	
2	(iii)		$\int_0^{\ln 3}[(e^x-2)^2-1]dx=\int_0^{\ln 3}[(e^x)^2-4e^x+4-1]dx$	M1	expanding brackets must have 3 terms: $(e^x)^2 - 4$ is M0, condone e^x^2	or if $u = e^x$, $\int_1^3[u^2-4u+4-1]/u\,du$
			$=\int_0^{\ln 3}[e^{2x}-4e^x+3]dx$	A1	$\int e^{2x} - 4e^x + 3$ [dx] (condone no dx)	$= \int u - 4 + 3/u \, du$
			$=\left[\frac{1}{2}e^{2x}-4e^x+3x\right]_0^{\ln 3}$	B1 A1ft	$\int e^{2x} = ½\, e^{2x}$ $[½\, e^{2x} - 4e^x + 3x]$	$= [\,½\, u^2 - 4u + 3\ln u]$
			$= (4.5 - 12 + 3\ln3) - (0.5 - 4)$			
			$= 3\ln3 - 4$ [so area $= 4 - 3\ln3$]	A1 **[5]**	condone $3\ln3 - 4$ as final ans; mark final ans	
2	(iv)		$y = (e^x - 2)^2 - 1 \quad x \leftrightarrow y$ $x = (e^y - 2)^2 - 1$			
			$\Rightarrow \quad x + 1 = (e^y - 2)^2$	M1	attempt to solve for y (might be indicated by expanding and then taking lns)	or x if x and y not interchanged yet or adding (or subtracting) 1
			$\Rightarrow \quad \pm\sqrt{(x + 1)} = e^y - 2$ (+ for $y \geq \ln 2$) $\Rightarrow \quad 2 + \sqrt{(x + 1)} = e^y$	A1	condone no ±	
			$\Rightarrow \quad y = \ln(2 + \sqrt{(x + 1)}) = f^{-1}(x)$	A1	must have interchanged x and y in final ans	
			Domain is $x \geq -1$	B1	must be ≥ and x (not y)	if not specified, assume first ans is domain and second range
			Range is $y \geq \ln 2$	B1	or $f^{-1}(x) \geq \ln 2$, must be ≥ (not x or $f(x)$) if $x > -1$ and $y > \ln 2$ SCB1	
				M1 A1 **[7]**	recognisable attempt to reflect curve, or any part of curve, in $y = x$ good shape, cross on $y = x$ (if shown), correct domain and range indicated. [see extra sheet for examples]	$y = x$ shown indicative but not essential e.g. −1 and ln 2 marked on axes

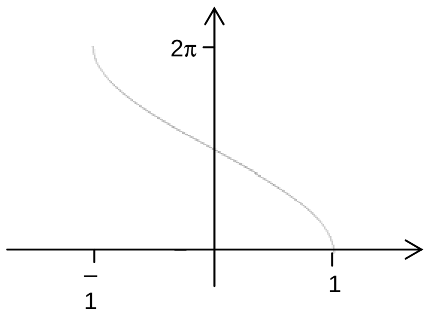
3 (i) bounds $-\pi + 1, \pi + 1$ $\Rightarrow$ $-\pi + 1 < f(x) < \pi + 1$	B1B1 B1cao [3]	or … $< y <$ … or $(-\pi + 1, \pi + 1)$	not … $< x <$ …, not 'between …'
(ii) $y = 2\arctan x + 1$ $x \leftrightarrow y$ $x = 2\arctan y + 1$	M1	attempt to invert formula	one step is enough, i.e. $y - 1 = 2\arctan x$ or $x - 1 = 2\arctan y$
$\Rightarrow$ $\frac{x-1}{2} = \arctan y$	A1	or $\frac{y-1}{2} = \arctan x$	need not have interchanged x and y at this stage
$\Rightarrow$ $y = \tan(\frac{x-1}{2}) \Rightarrow f^{-1}(x) = \tan(\frac{x-1}{2})$	A1		allow $y =$ …
	B1 B1 [5]	reasonable reflection in $y = x$ (1, 0) intercept indicated.	curves must cross on $y = x$ line if present (or close enough to imply intention) curves shouldn't touch or cross in the third quadrant

4 $gf(x) = e^{2\ln x}$ $= e^{\ln x^2}$ $= x^2$	M1 M1 A1 [3]	Forming gf(x) (soi)	Doing fg: $2\ln(e^x) = 2x$ SC1 Allow x^2 (but not $2x$) unsupported

5	$f(-x) = -f(x)$, $g(-x) = g(x)$ $g\,f(-x) = g\,[-f\,(x)]$ $= g\,f\,(x)$ $\Rightarrow$ g f is even	B1B1 M1 E1 [4]	condone f and g interchanged forming $gf(-x)$ or $gf(x)$ and using $f(-x) = -f(x)$ www

6	$f(x) = 1 + 2\sin 3x = y \quad x \leftrightarrow y$ $x = 1 + 2\sin 3y$ $\Rightarrow \sin 3y = (x - 1)/2$ $\Rightarrow 3y = \arcsin[(x - 1)/2]$ $\Rightarrow y = \frac{1}{3}\arcsin\left[\frac{x-1}{2}\right]$ so $f^{-1}(x) = \frac{1}{3}\arcsin\left[\frac{x-1}{2}\right]$ Range of f is -1 to 3 $\Rightarrow -1 \le x \le 3$	M1 A1 A1 A1 M1 A1 [6]	attempt to invert must be $y = \ldots$ or $f^{-1}(x) = \ldots$ or $-1 \le (x - 1)/2 \le 1$ must be 'x', not y or $f(x)$	at least one step attempted, or reasonable attempt at flow chart inversion (or any other variable provided same used on each side) condone <'s for M1 allow unsupported correct answers; -1 to 3 is M1 A0

7 *Either* $y = \frac{1}{2}\ln(x-1)$ $x \leftrightarrow y$ $\Rightarrow$ $x = \frac{1}{2}\ln(y-1)$ $\Rightarrow$ $2x = \ln(y-1)$ $\Rightarrow$ $e^{2x} = y - 1$ $\Rightarrow$ $1 + e^{2x} = y$ $\Rightarrow$ $g(x) = 1 + e^{2x}$	 M1 M1 E1	or $y = e^{(x-1)/2}$ attempt to invert and interchanging x with y o.e. (at any stage) $e^{\ln y-1} = y - 1$ or $\ln(e^y) = y$ used www
or $gf(x) = g(\frac{1}{2}\ln(x-1))$ $= 1 + e^{\ln(x-1)}$ $= 1 + x - 1$ $= x$	M1 M1 E1 [3]	or $fg(x) = \ldots$ (correct way round) $e^{\ln(x-1)} = x - 1$ or $\ln(e^{2x}) = 2x$ www

8 	M1 B1 A1 [3]	Can use degrees or radians reasonable shape (condone extra range) passes through $(-1, 2\pi)$, $(0, \pi)$ and $(1, 0)$ good sketches – look for curve reasonably vertical at $(-1, 2\pi)$ and $(1, 0)$, negative gradient at $(0, \pi)$. Domain and range must be clearly marked and correct.