1 **(i)** Show algebraically that the function $f(x) = \dfrac{2x}{1 - x^2}$ is odd. **[2]**

Fig. 7 shows the curve $y = f(x)$ for $0 \leqslant x \leqslant 4$, together with the asymptote $x = 1$.

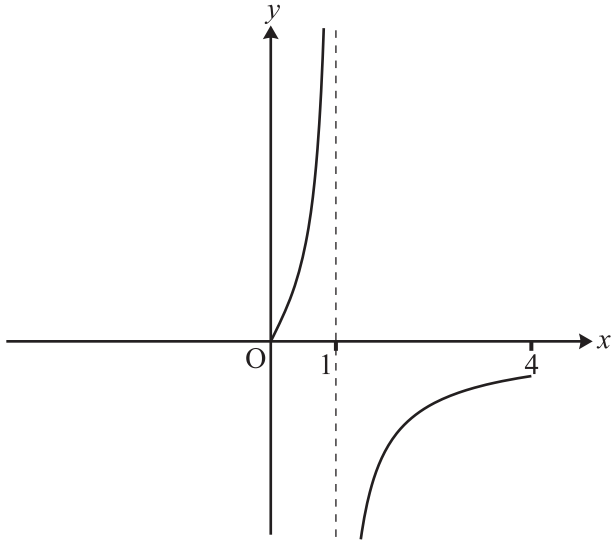


Fig. 7

(ii) Use the copy of Fig. 7 to complete the curve for $-4 \leqslant x \leqslant 4$. **[2]**

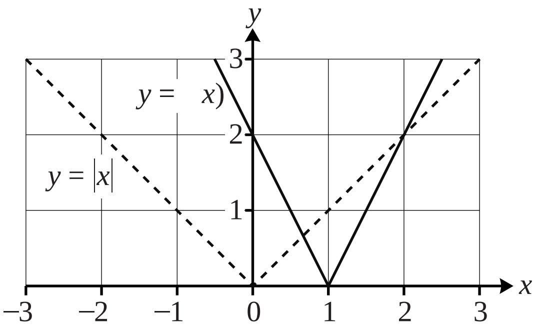
2 The functions f(x) and g(x) are defined as follows.

$$f(x) = \ln x, \quad x > 0$$
$$g(x) = 1 + x^2, \quad x \in \mathbb{R}$$

Write down the functions fg(x) and gf(x), and state whether these functions are odd, even or neither. **[4]**

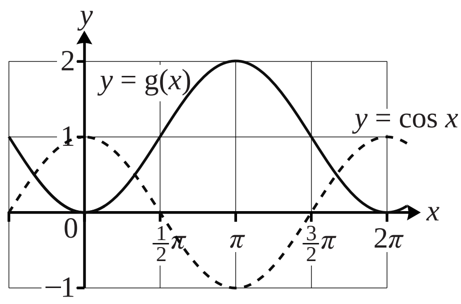
3 Each of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ below is obtained using a sequence of two transformations applied to the corresponding dashed graph. In each case, state suitable transformations, and hence find expressions for $\mathrm{f}(x)$ and $\mathrm{g}(x)$.

(i)



[3]

(ii)



[3]

4 Fig. 4 shows the curve $y = f(x)$, where $f(x) = \sqrt{1 - 9x^2}$, $-a \leqslant x \leqslant a$.

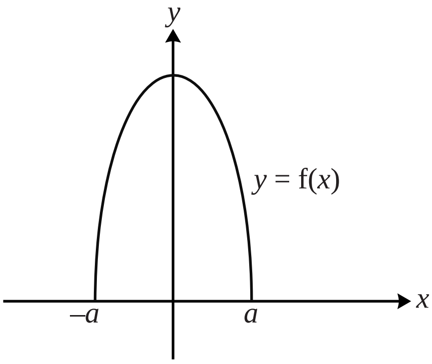


Fig. 4

(i) Find the value of a. **[2]**

(ii) Write down the range of $f(x)$. **[1]**

(iii) Sketch the curve $y = f(\frac{1}{3}x) - 1$. **[3]**

5 You are given that $f(x)$ and $g(x)$ are odd functions, defined for $x \in \mathbb{R}$.

(i) Given that $s(x) = f(x) + g(x)$, prove that $s(x)$ is an odd function. **[2]**

(ii) Given that $p(x) = f(x)g(x)$, determine whether $p(x)$ is odd, even or neither. **[2]**

6 **(i)** State the algebraic condition for the function $f(x)$ to be an even function.

What geometrical property does the graph of an even function have? [2]

(ii) State whether the following functions are odd, even or neither.

(A) $f(x) = x^2 - 3$

(B) $g(x) = \sin x + \cos x$

(C) $h(x) = \dfrac{1}{x + x^3}$ [3]

7 Fig. 8 shows part of the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \mathrm{e}^{-\frac{1}{5}x} \sin x$, for all x.

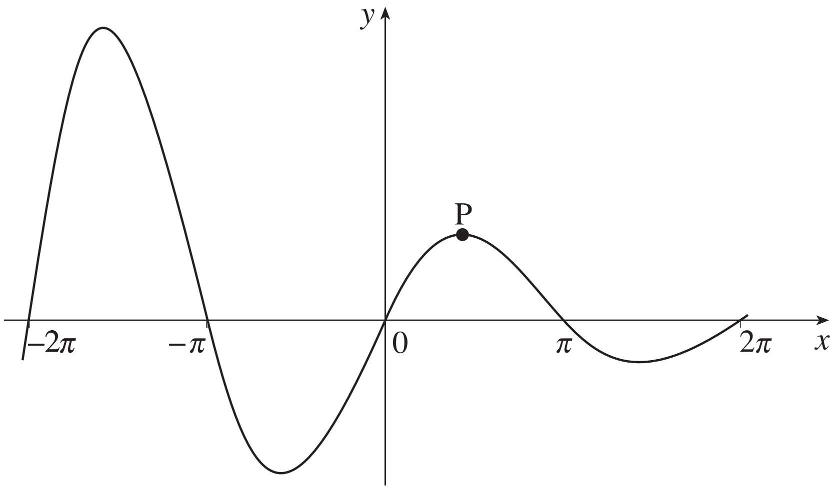


Fig. 8

(i) Sketch the graphs of

(*A*) $y = \mathrm{f}(2x)$,

(*B*) $y = \mathrm{f}(x + \pi)$. [4]

(ii) Show that the x-coordinate of the turning point P satisfies the equation $\tan x = 5$.

Hence find the coordinates of P. [6]

(iii) Show that $\mathrm{f}(x + \pi) = -\mathrm{e}^{-\frac{1}{5}\pi}\mathrm{f}(x)$. Hence, using the substitution $u = x - \pi$, show that

$$\int_{\pi}^{2\pi} \mathrm{f}(x)\,\mathrm{d}x = -\mathrm{e}^{-\frac{1}{5}\pi} \int_{0}^{\pi} \mathrm{f}(u)\,\mathrm{d}u.$$

Interpret this result graphically. [You should *not* attempt to integrate $\mathrm{f}(x)$.] [8]