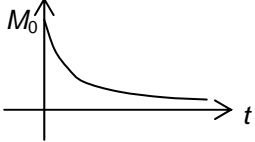


<p>1(i) $T = 25 + ae^{-kt}$. When $t = 0$, $T = 100$ $\Rightarrow 100 = 25 + ae^0$</p> <p>$\Rightarrow a = 75$ When $t = 3$, $T = 80$ $\Rightarrow 80 = 25 + 75e^{-3k}$ $\Rightarrow e^{-3k} = 55/75$ $\Rightarrow -3k = \ln(55/75)$, $k = -\ln(55/75) / 3 = 0.1034$</p>	<p>M1 A1 M1 M1 A1cao [5]</p>	<p>substituting $t = 0$ and $T = 100$ into their equation (even if this is an incorrect version of the given equation)</p> <p>substituting $t = 3$ and $T = 80$ into (their) equation taking lns correctly at any stage 0.1 or better or $-\frac{1}{3}\ln\left(\frac{55}{75}\right)$ o.e. if final answer</p>
<p>(ii) (A) $T = 25 + 75e^{-0.1034 \times 5} = 69.72$</p> <p>(B) 25°C</p>	<p>M1 A1 B1cao [3]</p>	<p>substituting $t = 5$ into their equation 69.5 to 70.5, condone inaccurate rounding due to value of k.</p>

<p>2 (i) When $t = 0$, $P = 5 + a = 8$ $\Rightarrow a = 3$ When $t = 1$, $5 + 3e^{-b} = 6$ $\Rightarrow e^{-b} = 1/3$ $\Rightarrow -b = \ln 1/3$ $\Rightarrow b = \ln 3 = 1.10$ (3 s.f.)</p> <p>(ii) 5 million</p>	<p>M1 A1 M1 M1 A1ft B1 [6]</p>	<p>substituting $t = 0$ into equation</p> <p>Forming equation using their a</p> <p>Taking lns on correct re-arrangement (ft their a)</p> <p>or $P = 5$</p>
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<p>3 (i) $\ln(3x^2)$</p> <p>(ii) $\ln 3x^2 = \ln(5x + 2)$ $\Rightarrow 3x^2 = 5x + 2$ $\Rightarrow 3x^2 - 5x - 2 = 0^*$</p> <p>(iii) $(3x + 1)(x - 2) = 0$ $\Rightarrow x = -1/3$ or 2</p> <p>$x = -1/3$ is not valid as $\ln(-1/3)$ is not defined</p>	<p>B1 B1 M1 E1 M1 A1cao B1ft [7]</p>	<p>$2\ln x = \ln x^2$ $\ln x^2 + \ln 3 = \ln 3x^2$</p> <p>Anti-logging</p> <p>Factorising or quadratic formula</p> <p>ft on one positive and one negative root</p>
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<p>4 (i)</p>  <p>(ii) $\frac{M}{M_0} = e^{-0.000121 \times 5730} = e^{-0.6933} \approx \frac{1}{2}$</p> <p>(iii) $\frac{M}{M_0} = e^{-kT} = \frac{1}{2}$ $\Rightarrow \ln \frac{1}{2} = -kT$ $\Rightarrow \ln 2 = kT$ $\Rightarrow T = \frac{\ln 2}{k}^*$</p> <p>(iv) $T = \frac{\ln 2}{2.88 \times 10^{-5}} \approx 24000$ years</p>	<p>B1 B1 M1 E1 M1 M1 E1 B1 [8]</p>	<p>Correct shape Passes through $(0, M_0)$</p> <p>substituting $k = -0.00121$ and $t = 5730$ into equation (or ln eqn) showing that $M \approx \frac{1}{2} M_0$</p> <p>substituting $M/M_0 = \frac{1}{2}$ into equation (oe) taking lns correctly</p> <p>24 000 or better</p>
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<p>5 $T = 30 + 20e^0 = 50$ $dT/dt = -0.05 \times 20e^{-0.05t} = -e^{-0.05t}$ When $t = 0$, $dT/dt = -1$</p> <p>When $T = 40$, $40 = 30 + 20e^{-0.05t}$ $\Rightarrow e^{-0.05t} = 1/2$ $\Rightarrow -0.05t = \ln 1/2$ $\Rightarrow t = -20 \ln 1/2 = 13.86..$ (mins)</p>	<p>B1 M1 A1cao</p> <p>M1 M1 A1cao</p> <p>[6]</p>	<p>50 correct derivative -1 (or 1)</p> <p>substituting $T = 40$ taking lns correctly or trial and improvement – one value above and one below or 13.9 or 13 mins 52 secs or better www condone secs</p>
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Question		Answer	Marks	Guidance
6	(i)	$\frac{dy}{dx} = \sin 2x + 2x \cos 2x$ $dy/dx = 0 \text{ when } \sin 2x + 2x \cos 2x = 0$ $\Rightarrow \frac{\sin 2x + 2x \cos 2x}{\cos 2x} = 0$ $\Rightarrow \tan 2x + 2x = 0 *$	M1 A1 M1 A1 [4]	d/dx(sin 2x) = 2cos 2x soi cao, mark final answer equating their derivative to zero, provided it has two terms must show evidence of division by cos 2x can be inferred from dy/dx = 2x cos 2x e.g. dy/dx = tan 2x + 2x is A0
	(ii)	At P, $x \sin 2x = 0$ $\Rightarrow \sin 2x = 0, 2x = (0), \pi \Rightarrow x = \pi/2$ At P, $dy/dx = \sin \pi + 2(\pi/2) \cos \pi = -\pi$ Eqn of tangent: $y - 0 = -\pi(x - \pi/2)$ $\Rightarrow y = -\pi x + \pi^2/2$ $\Rightarrow 2\pi x + 2y = \pi^2 *$ When $x = 0, y = \pi^2/2$, so Q is $(0, \pi^2/2)$	M1 A1 B1 ft M1 A1 M1A1 [7]	$x = \pi/2$ ft their $\pi/2$ and their derivative substituting 0, their $\pi/2$ and their $-\pi$ into $y - y_1 = m(x - x_1)$ NB AG can isw inexact answers from $\pi^2/2$ Finding $x = \pi/2$ using the given line equation is M0 or their $-\pi$ into $y = mx + c$, and then evaluating $c: y = (-\pi)x + c$, $0 = (-\pi)(\pi/2) + c$ M1 $\Rightarrow c = \pi^2/2$ $\Rightarrow y = -\pi x + \pi^2/2 \Rightarrow 2\pi x + 2y = \pi^2 *A1$
	(iii)	Area = triangle OPQ – area under curve Triangle OPQ = $\frac{1}{2} \times \pi/2 \times \pi^2/2 [= \pi^3/8]$ Parts: $u = x, dv/dx = \sin 2x$ $du/dx = 1, v = -\frac{1}{2} \cos 2x$ $\int_0^{\pi/2} x \sin 2x dx = \left[-\frac{1}{2} x \cos 2x \right]_0^{\pi/2} - \int_0^{\pi/2} -\frac{1}{2} \cos 2x dx$ $= \left[-\frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x \right]_0^{\pi/2}$ $= -\frac{1}{4} \pi \cos \pi + \frac{1}{4} \sin \pi - (-0 \cos 0 + \frac{1}{4} \sin 0) = \frac{1}{4} \pi [-0]$ So shaded area = $\pi^3/8 - \pi/4 = \pi(\pi^2 - 2)/8*$	M1 B1cao M1 A1ft A1 A1cao A1 [7]	soi (or area under PQ – area under curve allow art 3.9 condone $v = k \cos 2x$ soi ft their $v = -\frac{1}{2} \cos 2x$, ignore limits $[-\frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x]$ o.e., must be correct at this stage, ignore limits (so dep previous A1) NB AG must be from fully correct work area under line may be expressed in integral form or using integral: $\int_0^{\pi/2} \left(\frac{1}{2} \pi^2 - \pi x \right) dx = \left[\frac{1}{2} \pi^2 x - \frac{1}{2} \pi x^2 \right]_0^{\pi/2} = \frac{\pi^3}{4} - \frac{\pi^3}{8} [= \frac{\pi^3}{8}]$ v can be inferred from their 'uv'