

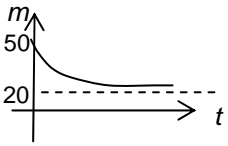
<p>1 $e^{2x} - 5e^x = 0$ $\Rightarrow e^x (e^x - 5) = 0$</p> <p>$\Rightarrow e^x = 5$ $\Rightarrow x = \ln 5$ or 1.6094</p>	<p>M1 M1 A1 A1 [4]</p>	<p>factoring out e^x or dividing $e^{2x} = 5e^x$ by e^x $e^{2x} / e^x = e^x$</p> <p>$\ln 5$ or 1.61 or better, mark final answer -1 for additional solutions, e.g. $x = 0$</p>
<p>or $\ln(e^{2x}) = \ln(5e^x)$ $\Rightarrow 2x = \ln 5 + x$ $\Rightarrow x = \ln 5$ or 1.6094</p>	<p>M1 A1 A1 A1 [4]</p>	<p>taking lns on $e^{2x} = 5e^x$ $2x, \ln 5 + x$ $\ln 5$ or 1.61 or better, mark final answer -1 for additional solutions, e.g. $x = 0$</p>

<p>2 (i) When $t = 0, T = 100$ $\Rightarrow 100 = 20 + b$ $\Rightarrow b = 80$ When $t = 5, T = 60$ $\Rightarrow 60 = 20 + 80 e^{-5k}$ $\Rightarrow e^{-5k} = 1/2$ $\Rightarrow k = \ln 2 / 5 = 0.139$</p>	<p>M1 A1</p> <p>M1 A1 [4]</p>	<p>substituting $t = 0, T = 100$ cao</p> <p>substituting $t = 5, T = 60$</p> <p>$1/5 \ln 2$ or 0.14 or better</p>
<p>(ii) $50 = 20 + 80 e^{-kt}$ $\Rightarrow e^{-kt} = 3/8$ $\Rightarrow t = \ln(8/3) / k = 7.075$ mins</p>	<p>M1 A1 [2]</p>	<p>Re-arranging and taking lns correctly – ft their b and k answers in range 7 to 7.1</p>

<p>3 $p = 100/x = 100 x^{-1}$ $\Rightarrow dp/dx = -100x^{-2} = -100/x^2$</p> <p>$dp/dt = dp/dx \times dx/dt$ $dx/dt = 10$</p> <p>When $x = 50$, $dp/dx = (-100/50^2)$ $\Rightarrow dp/dt = 10 \times -0.04 = -0.4$</p>	<p>M1 A1 M1 B1 M1dep A1cao [6]</p>	<p>attempt to differentiate $-100x^{-2}$ o.e. o.e. soi soi substituting $x = 50$ into their dp/dx dep 2nd M1 o.e. e.g. decreasing at 0.4</p>	<p>condone poor notation if chain rule correct or $x = 50 + 10 t$ B1 $\Rightarrow P = 100/x = 100/(50 + 10 t)$ $\Rightarrow dP/dt = -100(50 + 10 t)^{-2} \times 10 = -1000/(50 + 10t)^{-2}$ M1 A1 When $t = 0$, $dP/dt = -1000/50^2 = -0.4$ A1</p>
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<p>4(i) $100 = Ae^0 = A \Rightarrow A = 100$</p> <p>$50 = 100 e^{-1500k}$ $\Rightarrow e^{-1500k} = 0.5$ $\Rightarrow -1500k = \ln 0.5$ $\Rightarrow k = -\ln 0.5 / 1500 = 4.62 \times 10^{-4}$</p>	<p>M1A1 M1 M1 A1 [5]</p>	<p>$50 = A e^{-1500k}$ ft their 'A' if used</p> <p>taking lns correctly 0.00046 or better</p>
<p>(ii) $1 = 100e^{-kt}$ $\Rightarrow -kt = \ln 0.01$ $\Rightarrow t = -\ln 0.01 / k$ $= 9966$ years</p>	<p>M1 M1 A1 [3]</p>	<p>ft their A and k</p> <p>taking lns correctly art 9970</p>

<p>5 (i) When $n = 1$, $10\,000 = A e^b$ when $n = 2$, $16\,000 = A e^{2b}$ $\Rightarrow \frac{16000}{10000} = \frac{Ae^{2b}}{Ae^b} = e^b$ $\Rightarrow e^b = 1.6$ $\Rightarrow b = \ln 1.6 = 0.470$ $A = 10000/1.6 = 6250$.</p>	<p>B1 B1 M1 E1 B1 B1 [6]</p>	<p>soi soi eliminating A (do not allow verification) SCB2 if initial 'B's are missing, and ratio of years = 1.6 = e^b In 1.6 or 0.47 or better (mark final answer) cao – allow recovery from inexact b's</p>
<p>(ii) When $n = 20$, $P = 6250 \times e^{0.470 \times 20}$ = £75,550,000</p>	<p>M1 A1 [2]</p>	<p>substituting $n = 20$ into their equation with their A and b Allow answers from £75 000 000 to £76 000 000.</p>

<p>6(i) Initial mass = $20 + 30 e^0 = 50$ grams Long term mass = 20 grams</p>	<p>M1A1 B1 [3]</p>	
<p>(ii) $30 = 20 + 30 e^{-0.1t}$ $\Rightarrow e^{-0.1t} = 1/3$ $\Rightarrow -0.1t = \ln(1/3) = -1.0986\dots$ $\Rightarrow t = 11.0$ mins</p>	<p>M1 M1 A1 [3]</p>	<p>anti-logging correctly 11, 11.0, 10.99, 10.986 (not more than 3 d.p)</p>
<p>(iii)</p> 	<p>B1 B1 [2]</p>	<p>correct shape through (0, 50) – ignore negative values of t $\rightarrow 20$ as $t \rightarrow \infty$</p>

<p>7 (i) When $t = 0$, $V = 10\,000$ $\Rightarrow 10\,000 = Ae^0 = A$</p> <p>When $t = 3$, $V = 6000$ $\Rightarrow 6000 = 10\,000 e^{-3k}$ $\Rightarrow -3k = \ln(0.6) = -0.5108\dots$ $\Rightarrow k = 0.17(02\dots)$</p>	<p>M1 A1</p> <p>M1 M1</p> <p>A1 [5]</p>	<p>$10\,000 = Ae^0$ $A = 10\,000$</p> <p>taking lns (correctly) on their exponential equation - not logs unless to base 10 art 0.17 or $-(\ln 0.6)/3$ oe</p>
<p>(ii) $2000 = 10\,000e^{-kt}$ $\Rightarrow -kt = \ln 0.2$ $\Rightarrow t = -\ln 0.2 / k = 9.45$ (years)</p>	<p>M1</p> <p>A1 [2]</p>	<p>taking lns on correct equation (consistent with their k) allow art 9.5, but not 9.</p>