1 A cup of water is cooling. Its initial temperature is 100°C. After 3 minutes, its temperature is 80°C.

(i) Given that $T = 25 + ae^{-kt}$, where T is the temperature in °C, t is the time in minutes and a and k are constants, find the values of a and k. [5

(ii) What is the temperature of the water

(A) after 5 minutes,

(B) in the long term? [3]

2 A population is P million at time t years. P is modelled by the equation

$$P = 5 + ae^{-bt},$$

where a and b are constants.

The population is initially 8 million, and declines to 6 million after 1 year.

(i) Use this information to calculate the values of a and b, giving b correct to 3 significant figures. [5]

(ii) What is the long-term population predicted by the model? [1]

3 **(i)** Express $2\ln x + \ln 3$ as a single logarithm. [2]

(ii) Hence, given that x satisfies the equation

$$2\ln x + \ln 3 = \ln(5x + 2),$$

show that x is a root of the quadratic equation $3x^2 - 5x - 2 = 0$. [2]

(iii) Solve this quadratic equation, explaining why only one root is a valid solution of

$$2\ln x + \ln 3 = \ln(5x + 2).$$ [3]

4 The mass M kg of a radioactive material is modelled by the equation

$$M = M_0 e^{-kt},$$

where M_0 is the initial mass, t is the time in years, and k is a constant which measures the rate of radioactive decay.

(i) Sketch the graph of M against t. [2]

(ii) For Carbon 14, $k = 0.000\,121$. Verify that after 5730 years the mass M has reduced to approximately half the initial mass. [2]

The half-life of a radioactive material is the time taken for its mass to reduce to exactly half the initial mass.

(iii) Show that, in general, the half-life T is given by $T = \dfrac{\ln 2}{k}$. [3]

(iv) Hence find the half-life of Plutonium 239, given that for this material $k = 2.88 \times 10^{-5}$. [1]

5 The temperature T°C of a liquid at time t minutes is given by the equation

$$T = 30 + 20e^{-0.05t}, \quad \text{for } t \geqslant 0.$$

Write down the initial temperature of the liquid, and find the initial rate of change of temperature.

Find the time at which the temperature is 40 °C. [6]

6 Fig. 8 shows a sketch of part of the curve $y = x \sin 2x$, where x is in radians.

The curve crosses the x-axis at the point P. The tangent to the curve at P crosses the y-axis at Q.

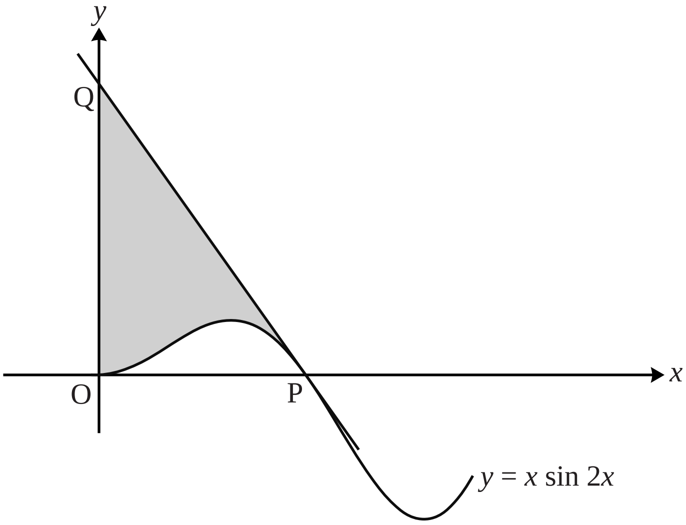

Fig. 8

(i) Find $\frac{dy}{dx}$. Hence show that the x-coordinates of the turning points of the curve satisfy the equation $\tan 2x + 2x = 0$. **[4]**

(ii) Find, in terms of π, the x-coordinate of the point P.

Show that the tangent PQ has equation $2\pi x + 2y = \pi^2$.

Find the exact coordinates of Q. **[7]**

(iii) Show that the exact value of the area shaded in Fig. 8 is $\frac{1}{8}\pi(\pi^2 - 2)$. **[7]**