1 The value £V of a car t years after it is new is modelled by the equation $V = Ae^{-kt}$, where A and k are positive constants which depend on the make and model of the car.

(i) Brian buys a new sports car. Its value is modelled by the equation

$$V = 20000\,e^{-0.2t}.$$

Calculate how much value, to the nearest £100, this car has lost after 1 year. **[2]**

(ii) At the same time as Brian buys his car, Kate buys a new hatchback for £15000. Her car loses £2000 of its value in the first year. Show that, for Kate's car, $k = 0.143$ correct to 3 significant figures. **[3]**

(iii) Find how long it is before Brian's and Kate's cars have the same value. **[3]**

2 The temperature θ °C of water in a container after t minutes is modelled by the equation

$$\theta = a - be^{-kt},$$

where a, b and k are positive constants.

The initial and long-term temperatures of the water are 15 °C and 100 °C respectively. After 1 minute, the temperature is 30 °C.

(i) Find a, b and k. **[6]**

(ii) Find how long it takes for the temperature to reach 80 °C. **[2]**

3 Oil is leaking into the sea from a pipeline, creating a circular oil slick. The radius r metres of the oil slick t hours after the start of the leak is modelled by the equation

$$r = 20(1 - e^{-0.2t}).$$

(i) Find the radius of the slick when $t = 2$, and the rate at which the radius is increasing at this time. **[4]**

(ii) Find the rate at which the area of the slick is increasing when $t = 2$. **[4]**

4 A termites' nest has a population of P million. P is modelled by the equation $P = 7 - 2e^{-kt}$, where t is in years, and k is a positive constant.

(i) Calculate the population when $t = 0$, and the long-term population, given by this model. **[3]**

(ii) Given that the population when $t = 1$ is estimated to be 5.5 million, calculate the value of k. **[3]**

5 The area of a circular stain is growing at a rate of 1 mm^2 per second. Find the rate of increase of its radius at an instant when its radius is 2 mm. **[5]**

6 **(i)** On a single set of axes, sketch the curves $y = e^x - 1$ and $y = 2e^{-x}$. **[3]**

(ii) Find the exact coordinates of the point of intersection of these curves. **[5]**

7 The height h metres of a tree after t years is modelled by the equation

$$h = a - be^{-kt},$$

where a, b and k are positive constants.

(i) Given that the long-term height of the tree is 10.5 metres, and the initial height is 0.5 metres, find the values of a and b. **[3]**

(ii) Given also that the tree grows to a height of 6 metres in 8 years, find the value of k, giving your answer correct to 2 decimal places. **[3]**