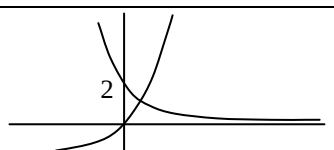
Question			Answer	Marks	Guidance	
1	(i)		$V = 20000e^{-0.2t}$ when $t = 1$, $V = 16374.615\ldots$	B1	(soi) art 16400	
			so car loses (£)3600	B1 **[2]**	condone no £, must be to nearest £100	or B2 for correct answer
	(ii)		When $t = 1$, $V = 13000$ $\Rightarrow$ $13000 = 15000\ e^{-k}$	M1		If $k = 0.143$ verified ,e.g. $15000\ e^{-0.143} = 13001[.31\ldots]$, SCB1
			$\Rightarrow$ $-k\ [\ln e] = \ln(13000/15000)$	M1	taking lns correctly oe e.g. $\ln 13000 = \ln 15000 - k\ [\ln e]$	need not have substituted for V and A
			$\Rightarrow k = 0.1431\ldots = 0.143$ (3sf) *	A1 **[3]**	cao **NB AG** must show some working if 4th d.p. not shown	e.g. $k = -\ln(13000/15000) = 0.143$
	(iii)		$15000e^{-0.143t} = 20000e^{-0.2t}$	M1*	must be correct, but could use a more accurate value for k	If M0, SCB1 for 5 – 5.1 years from correct calculations for each car, rot e.g. $t = 5$, £7358 (Brian), £7338(Kate) or (£7334 with more accurate k)
			$\Rightarrow$ $(15000/20000) = e^{(0.143 - 0.2)t}$	M1dep	dep *	o.e. e.g. $\ln 15000 - 0.143t = \ln 20000 - 0.2t$
			$\Rightarrow$ $t = \ln 0.75 / -0.057 = 5.05$ years so after 5 years	A1 **[3]**	cao accept answers in the range 5 − 5.1	

Question			Answer	Marks	Guidance	
2	(i)		$\theta = a - be^{-kt}$ When $t = 0$, $\theta = 15 \Rightarrow 15 = a - b$ When $t = \infty$, $\theta = 100 \Rightarrow 100 = a$ $\Rightarrow \quad b = 85$ When $t = 1$, $\theta = 30 \Rightarrow 30 = 100 - 85e^{-k}$ $\Rightarrow \quad e^{-k} = 70/85$ $\Rightarrow \quad -k = \ln(70/85) = -0.194(156\ldots)$ $\Rightarrow \quad k = 0.194$	 M1 B1 A1cao M1 M1 A1 **[6]**	 $1 = a - b$ $a = 100$ $b = 85$ $3 = a - b\,e^{-k}$ Re-arranging and taking lns 0.19 or better, or $-\ln(70/85)$ oe	 must have $e^0 = 1$ (need not substitute for a and b) allow $-k = \ln[(a - 30)/b]$ ft on a, b mark final ans
	(ii)		$80 = 100 - 85\,e^{-0.194t}$ $\Rightarrow \quad e^{-0.194t} = 20/85$ $\Rightarrow \quad t = -\ln(4/17) / 0.194 = 7.45$ (min)	M1 A1 **[2]**	ft their values for a, b and k art 7.5 or 7 min 30 s or better	but must substitute values

3	**(i)**		When $t = 2$, $r = 20(1 - e^{-0.4}) = 6.59$ m $dr/dt = -20 \times (-0.2e^{-0.2t})$ $= 4e^{-0.2t}$ When $t = 2$, $dr/dt = 2.68$	M1A1 M1 A1 **[4]**	6.6 or art 6.59 $-0.2e^{-0.2t}$ soi 2.7 or art 2.68 or $4e^{-0.4}$	 mark final answer
	(ii)		$A = \pi r^2$ $\Rightarrow \quad dA/dr = 2\pi r$ (= 41.428…) $dA/dt = (dA/dr) \times (dr/dt)$ $= 41.428… \times 2.68$ $= 111$ m^2/hr	M1 A1 M1 A1 **[4]**	attempt to differentiate πr^2 $dA/dr = 2\pi r$ (not dA/dt, dr/dA etc) (o.e.) chain rule expressed in terms of their A, r or implied 110 or art 111	or differentiating $400\pi(1 - e^{-0.2t})^2$ M1 $dA/dt = 400\pi.2(1 - e^{-0.2t}).(-0.2e^{-0.2t})$ A1 substitute $t = 2$ into correct dA/dt M1 (Could use another letter for A)

4	(i)		When $t = 0$, $P = 7 - 2 = 5$, so 5 (million) In the long term $e^{-kt} \rightarrow 0$ So long-term population is 7 (million)	B1 M1 A1 **[3]**	allow substituting a large number for t (for both marks)	allow 7 unsupported
	(ii)		$P = 7 - 2e^{-kt}$ When $t = 1$, $P = 5.5$ $\Rightarrow \quad 5.5 = 7 - 2e^{-k}$ $\Rightarrow \quad e^{-k} = (7 - 5.5)/2 = 0.75$ $\Rightarrow \quad -k = \ln((7 - 5.5)/2)$ $\Rightarrow \quad k = 0.288$ (3 s.f.)	 M1 M1 A1 **[3]**	 re-arranging and anti-logging – allow 1 slip (e.g. arith of $7 - 5.5$, or k for $-k$) or $\ln 2 - k = \ln 1.5$ o.e. 0.3 or better allow $\ln(4/3)$ or $-\ln(3/4)$ if final ans	 but penalise negative lns, e.g. $\ln(-1.5) = \ln(-2) - k$ rounding from a correct value of $k = 0.2876820725…$, penalise truncation, and incorrect work with negatives

5 $A = \pi$ $r^2 \Rightarrow$ When $r = 2$, $dA/dr = 4\pi$, $dA/dt = 1$ $dA/dr = \frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$ $2\pi r$ $\Rightarrow \quad 1 = 4\pi . dr/dt$ $\Rightarrow \quad dr/dt = 1/4\pi = 0.0796$ (mm/s)	M1A1 A1 M1 A1 [5]	$2\pi r$ soi (at any stage) chain rule (o.e) cao: 0.08 or better condone truncation	M1A0 if incorrect notation, e.g. dy/dx, dr/dA, if seen. $2r$ is M1A0 must be dA/dr (soi) and dA/dt any correct form stated with relevant variables , e.g. $\frac{dr}{dt} = \frac{dr}{dA} . \frac{dA}{dt}$, $\frac{dr}{dt} = \frac{dr}{dA} / \frac{dt}{dA}$, etc. allow $1/4\pi$ but mark final answer

6(i) 	B1 B1 B1 [3]	shape of $y = e^x - 1$ and through O shape of $y = 2e^{-x}$ through (0, 2) (not (2,0))	for first and second B1s graphs must include negative x values condone no asymptote $y = -1$ shown asymptotic to x-axis (shouldn't cross)
(ii) $e^x - 1 = 2e^{-x}$ $\Rightarrow \quad e^{2x} - e^x = 2$ $\Rightarrow \quad (e^x)^2 - e^x - 2 = 0$ $\Rightarrow \quad (e^x - 2)(e^x + 1) = 0$ $\Rightarrow \quad e^x = 2 \quad$ (or -1) $\Rightarrow \quad x = \ln 2$ $\Rightarrow \quad y = 1$	M1 M1 B1 B1 B1cao [5]	equating re-arranging into a quadratic in $e^x = 0$ stated www www www	 allow one error but must have $e^{2x} = (e^x)^2$ (soi) award even if not from quadratic method (i.e. by 'fitting') provided www allow for unsupported answers, provided www need not have used a quadratic, provided www

7(i)	$h = a - be^{-kt} \Rightarrow a = 10.5$ (their)$a - be^{0} = 0.5$ $\Rightarrow b = 10$	B1 M1 A1cao [3]	*a* need not be substituted	
(ii)	$h = 10.5 - 10e^{-kt}$ When $t = 8$, $h = 10.5 - 10e^{-8k} = 6$ $\Rightarrow$ $10e^{-8k} = 4.5$ $\Rightarrow$ $-8k = \ln 0.45$ $\Rightarrow$ $k = \ln 0.45/(-8) = 0.09981\ldots = 0.10$	 M1 M1 A1 [3]	 ft their *a* and *b* (even if made up) taking lns correctly on a correct re-arrangement - ft *a*, *b* if not eased cao (www) but allow 0.1	 allow M1 for $a - be^{-8k} = 6$ allow *a* and *b* unsubstituted allow their 0.45 (or 4.5) to be negative