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| 1 | | $h = r$ so $V = \pi h^3/3$ $dV/dt = 5$ | B1 B1 | o.e. e.g. $\pi h^3 \tan 45^\circ /3$ soi (can be implied from $V = 5t$) | e.g. from a correct chain rule |
| | | $dV/dh = \pi h^2$ $dV/dt = (dV/dh) \cdot dh/dt$ $\Rightarrow 5 = 100\pi dh/dt$ $\Rightarrow dh/dt = 5/100\pi = 0.016 \text{ cm s}^{-1}$ | B1ft M1 A1 | must be dV/dh soi, ft their $\pi h^3/3$ any correct chain rule in V , h and t (soi) 0.016 or better; accept $1/(20\pi)$ o.e., but mark final answer | but must have substituted for r e.g. $dh/dt = dh/dV \times dV/dt$, 0.01591549... penalise incorrect rounding |
| | | or $V = 5t$ so $\pi h^3/3 = 5t$ $\Rightarrow \pi h^2 dh/dt = 5$ $\Rightarrow dh/dt = 5/\pi h^2 = 5/100\pi = 0.016 \text{ cm s}^{-1}$ | B1 M1 A1 [5] | or $5 dt/dh = \pi h^2$ o.e. 0.016 or better; accept $1/(20\pi)$ o.e., but mark final answer | penalise incorrect rounding |

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| 2 | | $dV/dr = 4\pi r^2$ $dV/dt = 10$ $dV/dt = (dV/dr)(dr/dt)$ $\Rightarrow 10 = 4\pi \cdot 64 \cdot dr/dt$ $\Rightarrow dr/dt = 0.0124 \text{ cm s}^{-1}$ | B1 B1 M1 A1 A1 [5] | or $12\pi r^2/3$, condone dr/dV , dV/dR a correct chain rule soi o.e. (soi) must be correct 0.012 or better or $10/256\pi$ or $5/128\pi$ | Condone use of other letters for t o.e. e.g. $dr/dt = (dr/dV)(dV/dt)$ mark final answer |
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| 3 | (i) | $dF/dv = -25 v^{-2}$ | M1 A1 [2] | $d/dv(v^{-1}) = -v^{-2}$ soi $-25 v^{-2}$ o.e mark final ans | |
| | (ii) | When $v = 50$, $dF/dv = -25/50^2 (= -0.01)$ $\frac{dF}{dt} = \frac{dF}{dv} \cdot \frac{dv}{dt}$ $= -0.01 \times 1.5 = -0.015$ | B1 M1 Alcao [3] | $-25/50^2$ o. o.e. e.g. $-3/200$ isw | e.g. $\frac{dF}{dv} = \frac{dF}{dt} / \frac{dv}{dt}$ |

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| 4 | | $V = \pi h^2 \Rightarrow dV/dh = 2\pi h \Rightarrow$ $dV/dt = dV/dh \times dh/dt$ $dV/dt = 10$ $dh/dt = 10/(2\pi \times 5) = 1/\pi$ | M1A1 M1 B1 A1 [5] | if derivative $2\pi h$ seen without $dV/dh = \dots$ allow M1A0 soi ; o.e. – any correct statement of the chain rule using V , h and t – condone use of a letter other than t for time here soi; if a letter other than t used (and not defined) B0 or 0.32 or better, mark final answer | |
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| <p>5(i) $\frac{d}{dx} \left(\frac{\sin x}{\cos x} \right) = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$ $= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} *$</p> | <p>M1 A1 A1 [3]</p> | <p>Quotient (or product) rule (AG)</p> | <p>product rule: $\frac{1}{\cos x} \cdot \cos x + \sin x \left(-\frac{1}{\cos^2 x} \right) (-\sin x)$ but must show evidence of using chain rule on $1/\cos x$ (or $d/dx (\sec x) = \sec x \tan x$ used)</p> |
| <p>(ii) Area = $\int_0^{\pi/4} \frac{1}{\cos^2 x} dx$ $= [\tan x]_0^{\pi/4}$ $= \tan(\pi/4) - \tan 0 = 1$</p> | <p>B1 M1 A1 [3]</p> | <p>correct integral and limits (soi) [tan x] or $\left[\frac{\sin x}{\cos x} \right]$</p> | <p>condone no dx; limits can be implied from subsequent work unsupported scores M0</p> |
| <p>(iii) $f(0) = 1/\cos^2(0) = 1$ $g(x) = 1/2\cos^2(x + \pi/4)$ $g(0) = 1/2\cos^2(\pi/4) = 1$ $(\Rightarrow f \text{ and } g \text{ meet at } (0, 1))$</p> | <p>B1 M1 A1 [3]</p> | <p>must show evidence</p> | <p>or $f(\pi/4) = 1/\cos^2(\pi/4) = 2$ so $g(0) = 1/2 f(\pi/4) = 1$</p> |
| <p>(iv) Translation in x-direction through $-\pi/4$ Stretch in y-direction scale factor $1/2$</p> | <p>M1 A1 M1 A1 B1ft B1ft B1 B1dep [8]</p> | <p>must be in x-direction, or $\begin{pmatrix} -\pi/4 \\ 0 \end{pmatrix}$ must be in y-direction asymptotes correct min point $(-\pi/4, 1/2)$ curves intersect on y-axis correct curve, dep B3, with asymptote lines indicated and correct, and TP in correct position</p> | <p>‘shift’ or ‘move’ for ‘translation’ M1 A0; $\begin{pmatrix} -\pi/4 \\ 0 \end{pmatrix}$ alone SC1 ‘contract’ or ‘compress’ or ‘squeeze’ for ‘stretch’ M1A0; ‘enlarge’ M0 stated or on graph; condone no $x = \dots$, ft $\pi/4$ to right only (viz. $-\pi/4, 3\pi/4$) stated or on graph; ft $\pi/4$ to right only (viz. $(\pi/4, 1/2)$) ‘y-values halved’, or ‘x-values reduced by $\pi/4$, are M0 (not geometric transformations), but for M1 condone mention of x- and y- values provided transformation words are used.</p> |
| <p>(v) Same as area in (ii), but stretched by s.f. $1/2$. So area = $1/2$.</p> | <p>B1ft [1]</p> | <p>$1/2$ area in (ii)</p> | <p>or $\int_{-\pi/4}^0 g(x) dx = \frac{1}{2} \int_{-\pi/4}^0 \frac{1}{\cos^2(x + \pi/4)} dx = \frac{1}{2} [\tan(x + \pi/4)]_{-\pi/4}^0 = 1/2$ allow unsupported</p> |

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| 6 (i) $5 = k/100 \Rightarrow k = 500^*$ | E1 [1] | NB answer given |
| (ii) $\frac{dP}{dV} = -500V^{-2} = -\frac{500}{V^2}$ | M1 A1 [2] | $(-1)V^{-2}$ o.e. – allow $-k/V^2$ |
| (iii) $\frac{dP}{dt} = \frac{dP}{dV} \cdot \frac{dV}{dt}$ When $V = 100$, $dP/dV = -500/10000 = -0.05$ $dV/dt = 10$ $\Rightarrow dP/dt = -0.05 \times 10 = -0.5$ So P is decreasing at 0.5 Atm/s | M1 B1ft B1 A1 [4] | chain rule (any correct version) (soi) (soi) -0.5 ca |

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| 7 (i) $\frac{dV}{dt} = 2$ (ii) $\tan 30 = 1/\sqrt{3}$ $= r/h$ $\Rightarrow h = \sqrt{3} r$ $\Rightarrow V = \frac{1}{3}\pi r^2 \cdot \sqrt{3}r = \frac{\sqrt{3}}{3}\pi r^3^*$ $\frac{dV}{dr} = \sqrt{3}\pi r^2$ (iii) When $r = 2$, $dV/dr = 4\sqrt{3}\pi$ $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$ $\Rightarrow 2 = 4\sqrt{3}\pi dr/dt$ $\Rightarrow dr/dt = 1/(2\sqrt{3}\pi)$ or 0.092 cm s^{-1} | B1 M1 E1 B1 M1 M1 A1cao [7] | Correct relationship between r and h in any form From exact working only o.e. e.g. $(3\sqrt{3}/3)\pi r^2$ or $\frac{dr}{dt} = \frac{dr}{dV} \cdot \frac{dV}{dt}$ substituting 2 for dV/dt and $r = 2$ into their dV/dr |
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| <p>8(i) $V = \pi h^2 - \frac{1}{3}\pi h^3$ $\Rightarrow \frac{dV}{dh} = 2\pi h - \pi h^2$</p> <p>(ii) $\frac{dV}{dt} = 0.02$ $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$ $\Rightarrow \frac{dh}{dt} = \frac{0.02}{dV/dh} = \frac{0.02}{2\pi h - \pi h^2}$</p> <p>When $h = 0.4$, $\Rightarrow \frac{dh}{dt} = \frac{0.02}{0.8\pi - 0.16\pi} = 0.0099 \text{ m/min}$</p> | <p>M1 A1 B1 M1 M1dep A1cao [6]</p> | <p>expanding brackets (correctly) or product rule oe</p> <p>soi $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$ oe</p> <p>substituting $h = 0.4$ into their $\frac{dV}{dh}$ and $\frac{dV}{dt} = 0.02$ 0.01 or better or $1/32\pi$</p> |
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