1 Fig. 4 shows a cone with its axis vertical. The angle between the axis and the slant edge is 45°. Water is poured into the cone at a constant rate of $5\,\text{cm}^3$ per second. At time t seconds, the height of the water surface above the vertex O of the cone is h cm, and the volume of water in the cone is $V\,\text{cm}^3$.

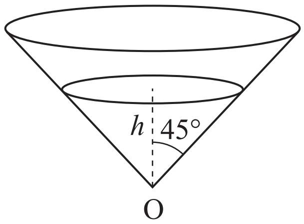

Fig. 4

Find V in terms of h.

Hence find the rate at which the height of water is increasing when the height is 10 cm.

[You are given that the volume V of a cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$]. **[5]**

2 A spherical balloon of radius r cm has volume $V\,\text{cm}^3$, where $V = \frac{4}{3}\pi r^3$. The balloon is inflated at a constant rate of $10\,\text{cm}^3\,\text{s}^{-1}$. Find the rate of increase of r when $r = 8$. **[5]**

3 The driving force F newtons and velocity v km s^{-1} of a car at time t seconds are related by the equation $F = \dfrac{25}{v}$.

(i) Find $\dfrac{\mathrm{d}F}{\mathrm{d}v}$. **[2]**

(ii) Find $\dfrac{\mathrm{d}F}{\mathrm{d}t}$ when $v = 50$ and $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 1.5$. **[3]**

4 Water flows into a bowl at a constant rate of $10\,\text{cm}^3\,\text{s}^{-1}$ (see Fig. 4).

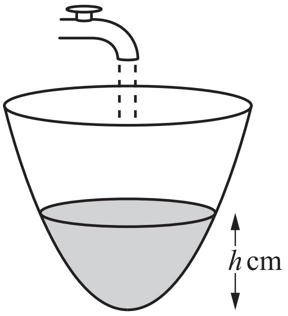

Fig. 4

When the depth of water in the bowl is h cm, the volume of water is V cm^3, where $V = \pi h^2$. Find the rate at which the depth is increasing at the instant in time when the depth is 5 cm. **[5]**

5 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{\cos^2 x}$, $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$, together with its asymptotes $x = \frac{1}{2}\pi$ and $x = -\frac{1}{2}\pi$.

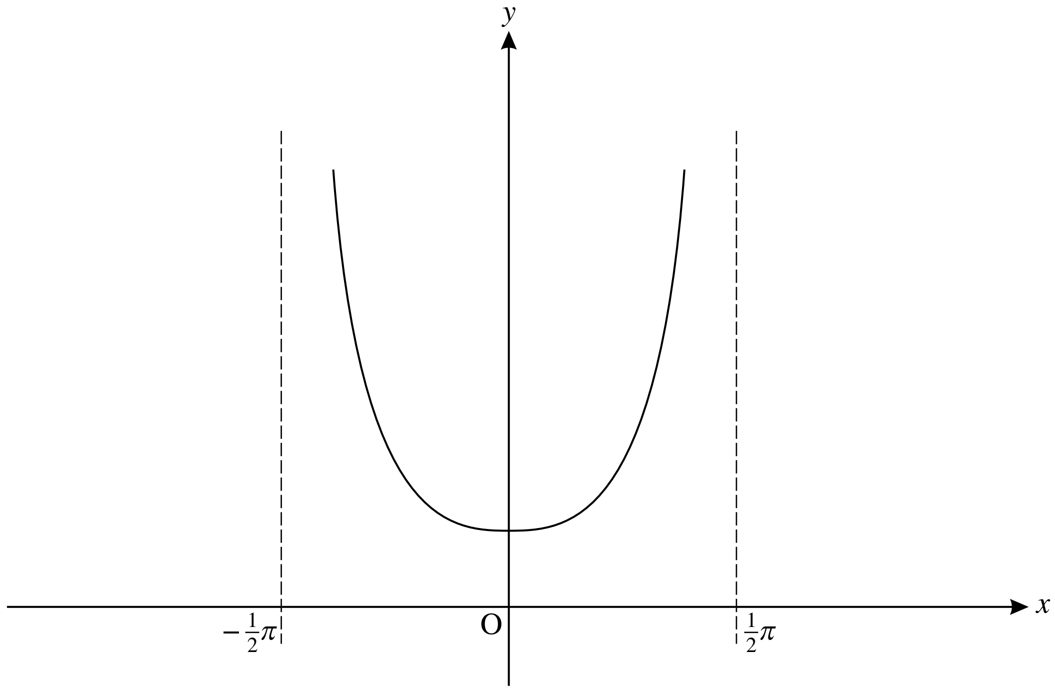

Fig. 9

(i) Use the quotient rule to show that the derivative of $\dfrac{\sin x}{\cos x}$ is $\dfrac{1}{\cos^2 x}$. **[3]**

(ii) Find the area bounded by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \frac{1}{4}\pi$. **[3]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \frac{1}{2}\mathrm{f}\left(x + \frac{1}{4}\pi\right)$.

(iii) Verify that the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ cross at $(0,\ 1)$. **[3]**

(iv) State a sequence of two transformations such that the curve $y = \mathrm{f}(x)$ is mapped to the curve $y = \mathrm{g}(x)$.

On the copy of Fig. 9, sketch the curve $y = \mathrm{g}(x)$, indicating clearly the coordinates of the minimum point and the equations of the asymptotes to the curve. **[8]**

(v) Use your result from part **(ii)** to write down the area bounded by the curve $y = \mathrm{g}(x)$, the x-axis, the y-axis and the line $x = -\frac{1}{4}\pi$. **[1]**

6 When the gas in a balloon is kept at a constant temperature, the pressure P in atmospheres and the volume $V\,\text{m}^3$ are related by the equation

$$P = \frac{k}{V},$$

where k is a constant. [This is known as Boyle's Law.]

When the volume is $100\,\text{m}^3$, the pressure is 5 atmospheres, and the volume is increasing at a rate of $10\,\text{m}^3$ per second.

(i) Show that $k = 500$. [1]

(ii) Find $\frac{dP}{dV}$ in terms of V. [2]

(iii) Find the rate at which the pressure is decreasing when $V = 100$. [4]

7 Fig. 4 shows a cone. The angle between the axis and the slant edge is 30°. Water is poured into the cone at a constant rate of $2\,\text{cm}^3$ per second. At time t seconds, the radius of the water surface is r cm and the volume of water in the cone is $V\,\text{cm}^3$.

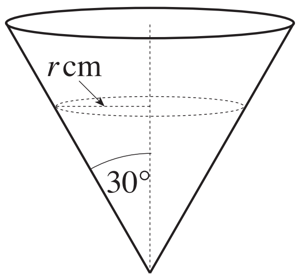

Fig. 4

(i) Write down the value of $\frac{dV}{dt}$. [1]

(ii) Show that $V = \frac{\sqrt{3}}{3}\pi r^3$, and find $\frac{dV}{dr}$. [3]

[You may assume that the volume of a cone of height h and radius r is $\frac{1}{3}\pi r^2 h$.]

(iii) Use the results of parts **(i)** and **(ii)** to find the value of $\frac{dr}{dt}$ when $r = 2$. [3]

8 Fig. 4 is a diagram of a garden pond.

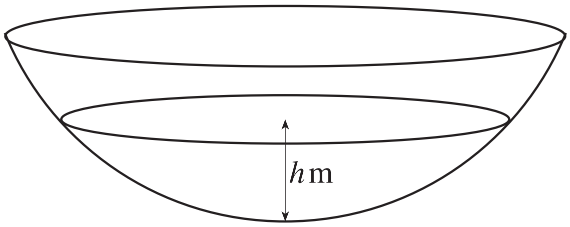

Fig. 4

The volume V m^3 of water in the pond when the depth is h metres is given by

$$V = \frac{1}{3}\pi h^2(3 - h).$$

(i) Find $\frac{dV}{dh}$. [2]

Water is poured into the pond at the rate of 0.02 m^3 per minute.

(ii) Find the value of $\frac{dh}{dt}$ when $h = 0.4$. [4]