1 The function $f(x)$ is defined by $f(x) = \sqrt{4 - x^2}$ for $-2 \leqslant x \leqslant 2$.

(i) Show that the curve $y = \sqrt{4 - x^2}$ is a semicircle of radius 2, and explain why it is not the whole of this circle. [3]

Fig. 9 shows a point P (a, b) on the semicircle. The tangent at P is shown.

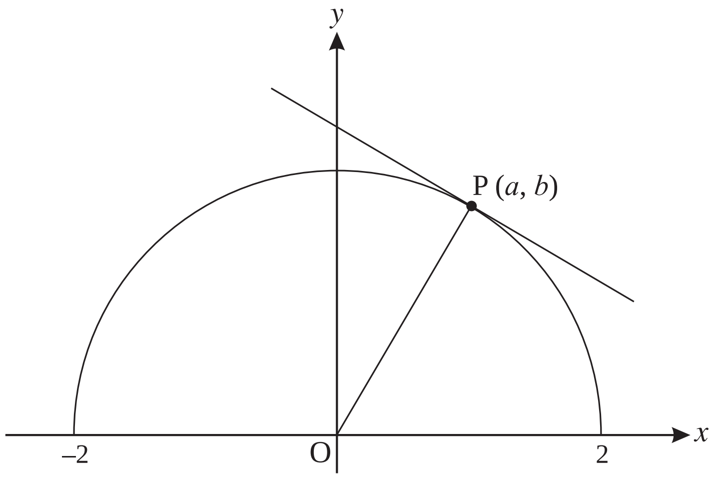

Fig. 9

(ii) (*A*) Use the gradient of OP to find the gradient of the tangent at P in terms of a and b.

(*B*) Differentiate $\sqrt{4 - x^2}$ and deduce the value of $f'(a)$.

(*C*) Show that your answers to parts (*A*) and (*B*) are equivalent. [6]

The function $g(x)$ is defined by $g(x) = 3f(x - 2)$, for $0 \leqslant x \leqslant 4$.

(iii) Describe a sequence of two transformations that would map the curve $y = f(x)$ onto the curve $y = g(x)$.

Hence sketch the curve $y = g(x)$. [6]

(iv) Show that if $y = g(x)$ then $9x^2 + y^2 = 36x$. [3]

2 Fig. 7 shows part of the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x\sqrt{1+x}$. The curve meets the x-axis at the origin and at the point P.

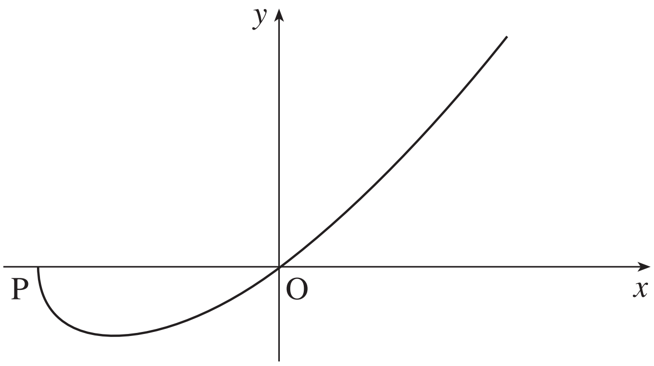

Fig. 7

(i) Verify that the point P has coordinates $(-1, 0)$. Hence state the domain of the function $\mathrm{f}(x)$. [2]

(ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2+3x}{2\sqrt{1+x}}$. [4]

(iii) Find the exact coordinates of the turning point of the curve. Hence write down the range of the function. [4]

(iv) Use the substitution $u = 1 + x$ to show that

$$\int_{-1}^{0} x\sqrt{1+x}\,\mathrm{d}x = \int_{0}^{1}\left(u^{\frac{3}{2}} \quad u^{\frac{1}{2}}\right)\mathrm{d}u.$$

Hence find the area of the region enclosed by the curve and the x-axis. [8]

3 Fig. 7 shows the curve $y = \frac{x^2}{1 + 2x^3}$. It is undefined at $x = a$; the line $x = a$ is a vertical asymptote.

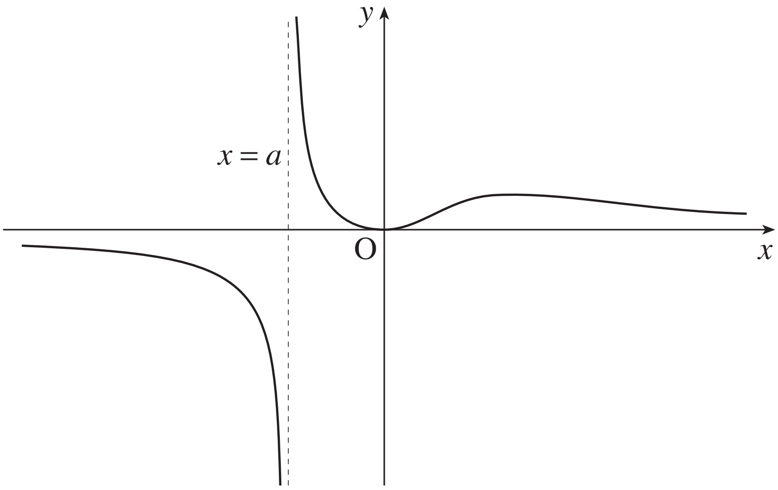

Fig. 7

(i) Calculate the value of a, giving your answer correct to 3 significant figures. [3]

(ii) Show that $\frac{dy}{dx} = \frac{2x - 2x^4}{(1 + 2x^3)^2}$. Hence determine the coordinates of the turning points of the curve. [8]

(iii) Show that the area of the region between the curve and the x-axis from $x = 0$ to $x = 1$ is $\frac{1}{6} \ln 3$. [5]