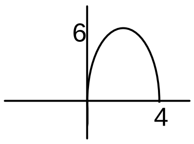
1 (i) $y=\sqrt{4-x^2}$ $\Rightarrow \quad y^2 = 4 - x^2$ $\Rightarrow \quad x^2 + y^2 = 4$ which is equation of a circle centre O radius 2 Square root does not give negative values, so this is only a semi-circle.	M1 A1 B1 [3]	squaring $x^2 + y^2 = 4$ + comment (correct) oe, e.g. f is a function and therefore single valued
(ii) (*A*) Grad of OP = b/a $\Rightarrow$ grad of tangent = $-\frac{a}{b}$ (*B*) $f'(x)=\frac{1}{2}(4-x^2)^{-1/2}.(-2x)$ $=-\frac{x}{\sqrt{4-x^2}}$ $\Rightarrow \quad f'(a)=-\frac{a}{\sqrt{4-a^2}}$ (*C*) $b = \sqrt{(4 - a^2)}$ so $f'(a)=-\frac{a}{b}$ as before	M1 A1 M1 A1 B1 E1 [6]	 chain rule or implicit differentiation oe substituting *a* into their f'(*x*)
(iii) Translation through $\begin{pmatrix}2\\0\end{pmatrix}$ followed by stretch scale factor 3 in *y*-direction 	M1 A1 M1 A1 M1 A1 [6]	Translation in *x*-direction through $\begin{pmatrix}2\\0\end{pmatrix}$ or 2 to right ('shift', 'move' M1 A0) $\begin{pmatrix}2\\0\end{pmatrix}$ alone is SC1 stretch in *y*-direction (condone *y* 'axis') (scale) factor 3 elliptical (or circular) shape through (0, 0) and (4, 0) and (2, 6) (soi) –1 if whole ellipse shown
(iv) $y = 3f(x - 2)$ $= 3\sqrt{(4 - (x - 2)^2)}$ $= 3\sqrt{(4 - x^2 + 4x - 4)}$ $= 3\sqrt{(4x - x^2)}$ $\Rightarrow \quad y^2 = 9(4x - x^2)$ $\Rightarrow \quad 9x^2 + y^2 = 36x$ *	M1 A1 E1 [3]	or substituting $3\sqrt{(4 - (x - 2)^2)}$ oe for *y* in $9x^2 + y^2$ $4x - x^2$ www

2(i) When $x = -1$, $y = -1\sqrt{0} = 0$ Domain $x \geq -1$	E1 B1 [2]	Not $y \geq -1$
(ii) $\frac{dy}{dx} = x.\frac{1}{2}(1+x)^{-1/2} + (1+x)^{1/2}$ $= ½\,(1 + x)^{-1/2}[x + 2(1 + x)]$ $= \frac{2+3x}{2\sqrt{1+x}}$ *	B1 B1 M1 E1	$x.\frac{1}{2}(1+x)^{-1/2}$ … + $(1 + x)^{1/2}$ taking out common factor or common denominator www
or $u = x + 1 \Rightarrow du/dx = 1$ $\Rightarrow y = (u - 1)u^{1/2} = u^{3/2} - u^{1/2}$ $\Rightarrow \frac{dy}{du} = \frac{3}{2}u^{\frac{1}{2}} - \frac{1}{2}u^{-\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{3}{2}(x+1)^{\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}$ $= \frac{1}{2}(x+1)^{-\frac{1}{2}}(3x+3-1)$ $= \frac{2+3x}{2\sqrt{1+x}}$ *	M1 A1 M1 E1 [4]	taking out common factor or common denominator
(iii) $dy/dx = 0$ when $3x + 2 = 0$ $\Rightarrow x = -2/3$ $\Rightarrow y = -\frac{2}{3}\sqrt{\frac{1}{3}}$ Range is $y \geq -\frac{2}{3}\sqrt{\frac{1}{3}}$	M1 A1cao A1 B1 ft [4]	o. not $x \geq -\frac{2}{3}\sqrt{\frac{1}{3}}$ (ft their y value, even if approximate)
(iv) $\int_{-1}^{0} x\sqrt{1+x}\,dx$ let $u = 1 + x$, $du/dx = 1 \Rightarrow du = dx$ when $x = -1$, $u = 0$, when $x = 0$, $u = 1$ $= \int_0^1 (u-1)\sqrt{u}\,du$ $= \int_0^1 (u^{3/2} - u^{1/2})du$ *	M1 B1 M1 E1	$du = dx$ or $du/dx = 1$ or $dx/du = 1$ changing limits – allow with no working shown provided limits are present and consistent with dx and du. $(u - 1)\sqrt{u}$ www – condone only final brackets missing, otherwise notation must be correct
$= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_0^1$ $= \pm\frac{4}{15}$	B1 B1 M1 A1cao [8]	$\frac{2}{5}u^{5/2}, -\frac{2}{3}u^{3/2}$ (oe) substituting correct limits (can imply the zero limit) $\pm\frac{4}{15}$ or ± 0.27 or better, not 0.26

3(i) Asymptote when $1 + 2x^3 = 0$ $\Rightarrow 2x^3 = -1$ $\Rightarrow x = -\frac{1}{\sqrt[3]{2}}$ $= -0.794$	M1 A1 A1cao [3]	oe, condone $\pm\frac{1}{\sqrt[3]{2}}$ if positive root is rejected must be to 3 s f.
(ii) $\frac{dy}{dx} = \frac{(1+2x^3).2x - x^2.6x^2}{(1+2x^3)^2}$ $= \frac{2x+4x^4-6x^4}{(1+2x^3)^2}$ $= \frac{2x-2x^4}{(1+2x^3)^2}$ * $dy/dx = 0$ when $2x(1 - x^3) = 0$ $\Rightarrow x = 0, y = 0$ or $x = 1$, $y = 1/3$	M1 A1 E1 M1 B1 B1 B1 B1 [8]	Quotient or product rule: (udv–vdu M0) $2x(1+2x^3)^{-1} + x^2(-1)(1+2x^3)^{-2}.6x^2$ allow one slip on derivatives correct expression – condone missing bracket if if intention implied by following line derivative = 0 x = 0 or 1 – allow unsupported answers y = 0 and 1/3 SC–1 for setting denom = 0 or extra solutions (e.g. x = –1)
(iii) $A = \int_0^1 \frac{x^2}{1+2x^3}\,dx$	M1	Correct integral and limits – allow $\int_1^0$
either $= \left[\frac{1}{6}\ln(1+2x^3)\right]_0^1$ $= \frac{1}{6}\ln 3$ *	M1 A1 M1 E1	$k\ln(1 + 2x^3)$ $k = 1/6$ substituting limits dep previous M1 www
or let $u = 1 + 2x^3 \Rightarrow du = 6x^2dx$ $\Rightarrow A = \int_1^3 \frac{1}{6}.\frac{1}{u}\,du$ $= \left[\frac{1}{6}\ln u\right]_1^3$ $= \frac{1}{6}\ln 3$ *	M1 A1 M1 E1 [5]	$\frac{1}{6u}$ $\frac{1}{6}\ln u$ substituting correct limits (but must have used substitution) www