1	**(i)**		$f'(x) = \dfrac{x.2(x-2)-(x-2)^2}{x^2}$	M1	quotient (or product) rule, condone sign errors only	e.g. $\dfrac{\pm x.2(x-2) \pm (x-2)^2}{x^2}$
				A1	correct exp, condone missing brackets here	PR: $(x - 2)^2.(-x^{-2}) + (1/x).2(x-2)$
			$= \dfrac{2x^2-4x-x^2+4x-4}{x^2}$			
			$= (x^2 - 4)/x^2 = 1 - 4/x^2$ *	A1	simplified correctly **NB AG**	with correct use of brackets
			or $f(x) = (x^2 - 4x + 4)/x$			
			$= x - 4 + 4/x$	M1	expanding bracket and dividing each term by x	must be 3 terms: $(x^2 - 4)/x$ is M0
				A1	correctly	e.g. $x - 4 + 2/x$ is M1A0
			$\Rightarrow f'(x) = 1 - 4/x^2$ *	A1	not from wrong working **NB AG**	
			$f''(x) = 8/x^3$	B1	o.e. e.g. $8x^{-3}$ or $8x/x^4$	
			$f'(x) = 0$ when $x^2 = 4$, $x = \pm 2$	M1	$x = \pm 2$ found from $1 - 4/x^2 = 0$	allow for $x = -2$ unsupported
			so at Q, $x = -2$, $y = -8$.	A1	$(-2, -8)$	
			$f''(-2)$ [$= -1$] < 0 so maximum	B1dep **[7]**	dep first B1. Can omit -1, but if shown must be correct. Must state < 0 or negative.	must use 2nd derivative test

Question			Answer	Marks	Guidance	
1	(ii)		$f(1) = (-1)^2/1 = 1$ $f(4) = (2)^2/4 = 1$	B1	verifying f(1) = 1 and f(4) = 1	or $(x-2)^2 = x \Rightarrow x^2 - 5x + 4 = 0$ $\Rightarrow (x-1)(x-4) = 0, x = 1, 4$
			$\int_1^4 \frac{(x-2)^2}{x}\,dx = \int_1^4 (x - 4 + 4/x)dx$	M1	expanding bracket and dividing each term by x 3 terms: $x - 4/x$ is M0	if $u = x - 2$ $\int \frac{u^2}{u+2}du = \int (u - 2 + \frac{4}{u+2})du$
			$= \left[x^2/2 - 4x + 4\ln x\right]_1^4$	A1	$x^2/2 - 4x + 4 \ln x$	$u^2/2 - 2u + 4 \ln (u + 2)$
			$= (8 - 16 + 4\ln 4) - (½ - 4 + 4\ln 1)$			
			$= 4\ln 4 - 4½$	A1cao		
			Area enclosed = rectangle – curve	M1	soi	
			$= 3 \times 1 - (4\ln 4 - 4½) = 7½ - 4\ln 4$	A1cao	o.e. but must combine numerical terms and evaluate ln 1 – mark final ans	
			or			
			Area = $\int_1^4 [1 - \frac{(x-2)^2}{x}]dx$	M1	no need to have limits	
			$= \int_1^4 (5 - x - 4/x)dx$	M1 A1	expanding bracket and dividing each term by x $5 - x - 4/x$	must be 3 terms in $(x-2)^2$ expansion
			$= \left[5x - x^2/2 - 4\ln x\right]_1^4$	A1	$5x - x^2/2 - 4 \ln x$	
			$= 20 - 8 - 4\ln 4 - (5 - ½ - 4\ln 1)$ $= 7½ - 4\ln 4$	A1cao **[6]**	o.e. but must combine numerical terms and evaluate ln 1 – mark final ans	
	(iii)		$[g(x) =]\ f(x+1) - 1$	M1	soi [may not be stated]	
			$= \frac{(x+1-2)^2}{x+1} - 1$	A1		
			$= \frac{x^2 - 2x + 1 - x - 1}{x+1} = \frac{x^2 - 3x}{x+1}$ *	A1 **[3]**	correctly simplified – not from wrong working **NB AG**	

Question			Answer	Marks	Guidance	
1	**(iv)**		Area is the same as that found in part (ii)	M1	award M1 for ± ans to 8(ii) (unless zero)	
			4ln4 – 7½	A1cao **[2]**	need not justify the change of sign	

2	**(i)**		$xe^{-2x} = mx$	M1	may be implied from 2nd line	
			$\Rightarrow e^{-2x} = m$	M1	dividing by x, or subtracting ln x	o.e. e.g. $[\ln x] - 2x = \ln m + [\ln x]$ or factorising: $x(e^{-2x} - m) = 0$
			$\Rightarrow -2x = \ln m$			
			$\Rightarrow x = -½ \ln m$ *	A1	**NB AG**	
			or			
			If $x = -½ \ln m$, $y = -½ \ln m \times e^{\ln m}$	M1	substituting correctly	
			$= -½ \ln m \times m$	A1		
			so P lies on $y = mx$	A1 **[3]**		
	(ii)		let $u = x$, $u' = 1$, $v = e^{-2x}$, $v' = -2e^{-2x}$	M1*	product rule consistent with their derivs	
			$dy/dx = e^{-2x} - 2xe^{-2x}$	A1	o.e. correct expression	
			$= e^{-2(-\frac{1}{2}\ln m)} - 2.(-\frac{1}{2}\ln m)e^{-2(-\frac{1}{2}\ln m)}$	M1dep	subst $x = -½ \ln m$ into their deriv dep M1*	
			$= e^{\ln m} + e^{\ln m} \ln m$ [$= m + m \ln m$]	A1cao **[4]**	condone $e^{\ln m}$ not simplified	but not $-2(-½ \ln m)$, but mark final ans

Question			Answer	Marks	Guidance	
2	(iii)		$m + m\ln m = -m$ $\Rightarrow \quad \ln m = -2$ $\Rightarrow \quad m = e^{-2}$ *	M1 A1	their gradient from (ii) $= -m$ **NB AG**	
			or $y + ½m\ln m = m(1 + \ln m)(x + ½\ln m)$ $x = -\ln m$, $y = 0 \Rightarrow ½m\ln m = m(1+\ln m)(-½\ln m)$ $\Rightarrow 1 + \ln m = -1$, $\ln m = -2$, $m = e^{-2}$	B2	for fully correct methods finding x-intercept of equation of tangent and equating to $-\ln m$	
			At P, $x = 1$	B1		
			$\Rightarrow \quad y = e^{-2}$	B1 **[4]**	isw approximations	not $e^{-2} \times 1$
	(iv)		Area under curve $= \int_0^1 xe^{-2x}dx$			
			$u = x,\ u' = 1,\ v' = e^{-2x},\ v = -½\, e^{-2x}$	M1	parts, condone $v = k\, e^{-2x}$, provided it is used consistently in their parts formula	ignore limits until 3rd A1
			$= \left[-\frac{1}{2}xe^{-2x}\right]_0^1 + \int_0^1 \frac{1}{2}e^{-2x}dx$	A1ft	ft their v	
			$= \left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^1$	A1	$-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$ o.e	
			$= (-½\, e^{-2} - ¼\, e^{-2}) - (0 - ¼\, e^{0})$ $[= ¼ - ¾\, e^{-2}]$	A1	correct expression	need not be simplified
			Area of triangle = ½ base × height	M1	ft their 1, e^{-2} or $[e^{-2}x^2/2]$	o.e. using isosceles triangle
			$= ½ \times 1 \times e^{-2}$	A1		M1 may be implied from 0.067…
			So area enclosed $= ¼ - 5e^{-2}/4$	A1cao **[7]**	o.e. must be exact, two terms only	isw

3(i) $\int_0^1 \frac{x^3}{1+x}\,dx$ let $u = 1+x$, $du = dx$ when $x = 0$, $u = 1$, when $x = 1$, $u = 2$ $= \int_1^2 \frac{(u-1)^3}{u}\,du$ $= \int_1^2 \frac{(u^3-3u^2+3u-1)}{u}\,du$ $= \int_1^2 (u^2-3u+3-\frac{1}{u})\,du$ * $\int_0^1 \frac{x^3}{1+x}\,dx = \left[\frac{1}{3}u^3-\frac{3}{2}u^2+3u-\ln u\right]_1^2$ $= (\frac{8}{3}-6+6-\ln 2)-(\frac{1}{3}-\frac{3}{2}+3-\ln 1)$ $= \frac{5}{6}-\ln 2$	B1 B1 M1 A1dep B1 M1 A1cao [7]	$a = 1$, $b = 2$ $(u-1)^3/u$ expanding (correctly) dep du = dx (o.e.) **AG** $\left[\frac{1}{3}u^3-\frac{3}{2}u^2+3u-\ln u\right]$ substituting correct limits dep integrated must be exact – must be 5/6	seen anywhere, e.g. in new limits e.g. du/dx = 1, condone missing dx's and du's, allow du = 1 upper – lower; may be implied from 0.140… must have evaluated ln 1 = 0
(ii) $y = x^2 \ln(1+x)$ $\Rightarrow$ $\frac{dy}{dx} = x^2.\frac{1}{1+x}+2x.\ln(1+x)$ $= \frac{x^2}{1+x}+2x\ln(1+x)$ When $x = 0$, dy/dx = 0 + 0.ln 1 = 0 ($\Rightarrow$ Origin is a stationary point)	M1 B1 A1 M1 A1cao [5]	Product rule d/dx (ln(1 + x)) = 1/(1 + x) cao (oe) mark final ans substituting x = 0 into **correct** deriv www	 or d/dx (ln u) = 1/u where u = 1 + x ln1+x is A0 when x = 0, dy/dx = 0 with no evidence of substituting M1A0 but condone missing bracket in ln(1+x)
(iii) $A = \int_0^1 x^2 \ln(1+x)\,dx$ let $u = \ln(1+x)$, $dv/dx = x^2$ $\frac{du}{dx} = \frac{1}{1+x}$, $v = \frac{1}{3}x^3$ $\Rightarrow$ $A = \left[\frac{1}{3}x^3\ln(1+x)\right]_0^1 - \int_0^1 \frac{1}{3}\frac{x^3}{1+x}\,dx$ $= \frac{1}{3}\ln 2-(\frac{5}{18}-\frac{1}{3}\ln 2)$ $= \frac{1}{3}\ln 2-\frac{5}{18}+\frac{1}{3}\ln 2$ $= \frac{2}{3}\ln 2-\frac{5}{18}$	B1 M1 A1 B1 B1ft A1 [6]	Correct integral and limits parts correct $= \frac{1}{3}\ln 2 - ...$ … – 1/3 (result from part (i)) cao	condone no dx, limits (and integral) can be implied by subsequent work u, du/dx, dv/dx and v all correct (oe) condone missing brackets condone missing bracket, can re-work from scratch oe e.g. $= \frac{12\ln 2-5}{18}, \frac{1}{3}\ln 4-\frac{5}{18}$, etc but must have evaluated ln 1 =0 Must combine the two ln terms