1 Fig. 9 shows the curve $y = \dfrac{x^2}{3x-1}$.

P is a turning point, and the curve has a vertical asymptote $x = a$.

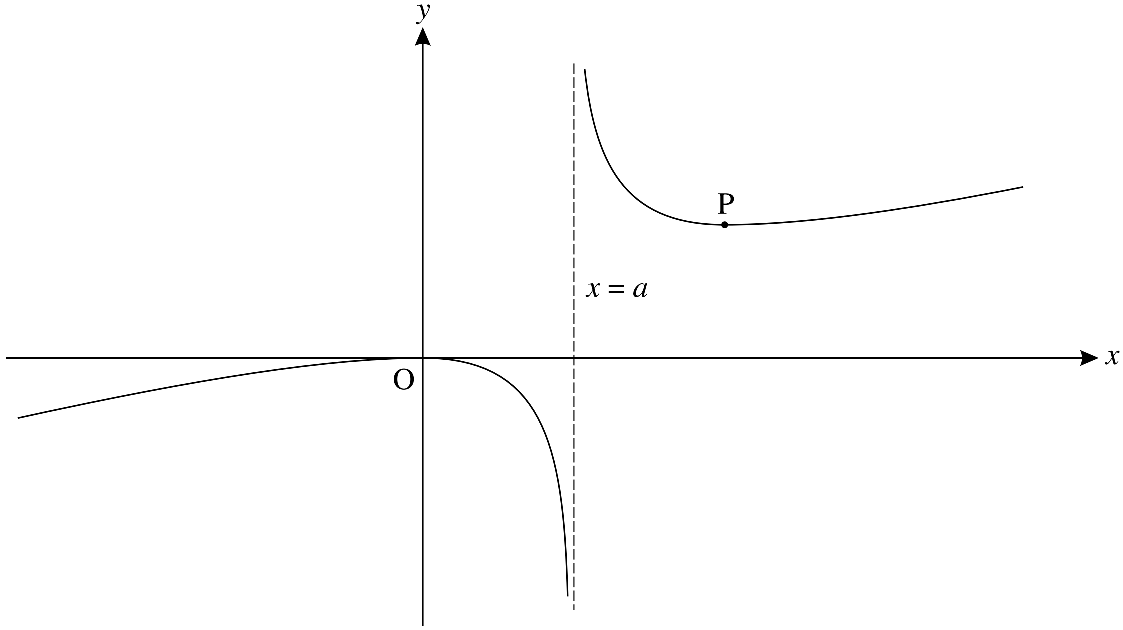

Fig. 9

(i) Write down the value of a. **[1]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{x(3x-2)}{(3x-1)^2}$. **[3]**

(iii) Find the exact coordinates of the turning point P.

Calculate the gradient of the curve when $x = 0.6$ and $x = 0.8$, and hence verify that P is a minimum point. **[7]**

(iv) Using the substitution $u = 3x - 1$, show that $\displaystyle\int \frac{x^2}{3x-1}\,dx = \frac{1}{27}\int \left(u + 2 + \frac{1}{u}\right) du$.

Hence find the exact area of the region enclosed by the curve, the x-axis and the lines $x = \frac{2}{3}$ and $x = 1$. **[7]**

2 Differentiate $\sqrt[3]{1+6x^2}$. [4]

3 Show that the curve $y = x^2 \ln x$ has a stationary point when $x = \frac{1}{\sqrt{e}}$. [6]

4 The equation of a curve is $y = \frac{x^2}{2x+1}$.

(i) Show that $\frac{dy}{dx} = \frac{2x(x+1)}{(2x+1)^2}$. [4]

(ii) Find the coordinates of the stationary points of the curve. You need not determine their nature. [4]

5 **(i)** Differentiate $\sqrt{1+2x}$.

(ii) Show that the derivative of $\ln(1 - e^{-x})$ is $\frac{1}{e^x - 1}$. [4]

6 The function $f(x) = \dfrac{\sin x}{2 - \cos x}$ has domain $-\pi \leqslant x \leqslant \pi$.

Fig. 8 shows the graph of $y = f(x)$ for $0 \leqslant x \leqslant \pi$.

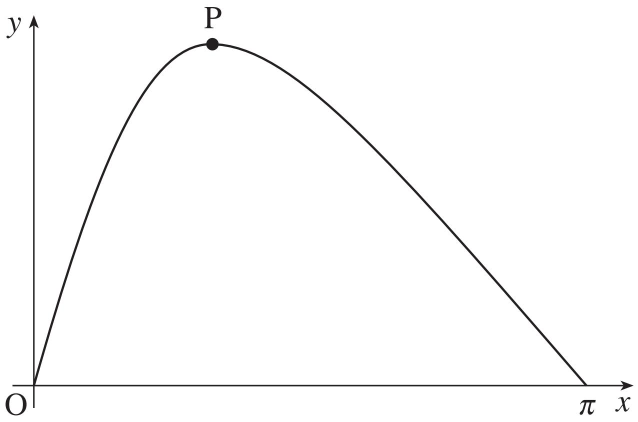

Fig. 8

(i) Find $f(-x)$ in terms of $f(x)$. Hence sketch the graph of $y = f(x)$ for the complete domain $-\pi \leqslant x \leqslant \pi$. [3]

(ii) Show that $f'(x) = \dfrac{2\cos x - 1}{(2 - \cos x)^2}$. Hence find the exact coordinates of the turning point P.

State the range of the function $f(x)$, giving your answer exactly. [8]

(iii) Using the substitution $u = 2 - \cos x$ or otherwise, find the exact value of $\displaystyle\int_0^{\pi} \frac{\sin x}{2 \quad \cos x}\,dx$. [4]

(iv) Sketch the graph of $y = f(2x)$. [1]

(v) Using your answers to parts **(iii)** and **(iv)**, write down the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{2 \quad \cos 2x}\,dx$. [2]

7 Fig. 3 shows the curve defined by the equation $y = \arcsin(x - 1)$, for $0 \leqslant x \leqslant 2$.

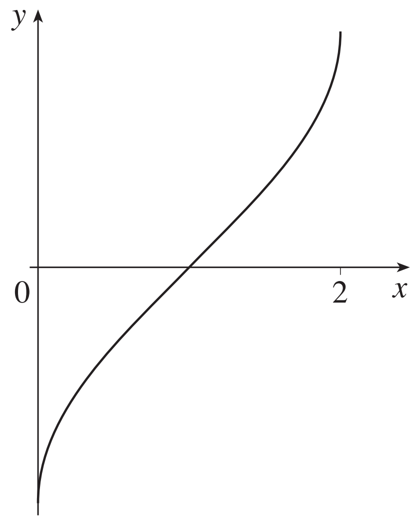

Fig. 3

(i) Find x in terms of y, and show that $\frac{dx}{dy} = \cos y$. [3]

(ii) Hence find the exact gradient of the curve at the point where $x = 1.5$. [4]

8 A curve has equation $y = \frac{x}{2 + 3\ln x}$. Find $\frac{dy}{dx}$. Hence find the exact coordinates of the stationary point of the curve. [7]