

| Question | | Answer | Marks | Guidance | |
|------------------------|-------|--|-------|---|---|
| 1 | | $y = e^{2x} \cos x$ | M1 | product rule used | consistent with their derivs e.g. $2e^{2x} - e^{2x} \tan x$ is A0 or $\sin^2 x + \cos^2 x = 1$ used 1.1071487 ..., 0.352416... π , penalise incorrect rounding no choice |
| | | $\Rightarrow \frac{dy}{dx} = 2e^{2x} \cos x - e^{2x} \sin x$ | A1 | cao – mark final ans | |
| | | $\frac{dy}{dx} = 0 \Rightarrow e^{2x}(2\cos x - \sin x) = 0$ | M1 | their derivative = 0 | |
| | | $\Rightarrow 2\cos x = \sin x$ | | | |
| | | $\Rightarrow 2 = \sin x / \cos x = \tan x$ | M1 | $\sin x / \cos x = \tan x$ used | |
| | | $\Rightarrow x = 1.11$ | A1 | 1.1 or 0.35π or better, or $\arctan 2$, not 63.4° but condone ans given in both degrees and radians here | |
| $\Rightarrow y = 4.09$ | A1cao | art 4.1 | | | |
| | | | [6] | | |

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| 2 | | $y = \ln(1 - \cos 2x)$, let $u = 1 - \cos 2x$ | M1 | $1/(1 - \cos 2x)$ soi | must be in at least two places isw after correct answer seen |
| | | $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ | | | |
| | | $= (1/u) \cdot 2\sin 2x$ | M1 | $d/dx (1 - \cos 2x) = \pm 2\sin 2x$ | |
| | | $= \frac{2\sin 2x}{1 - \cos 2x}$ | A1cao | | |
| | | When $x = \pi/6$, $\frac{dy}{dx} = \frac{2\sin(\pi/3)}{1 - \cos(\pi/3)}$ | M1 | substituting $\pi/6$ or 30° into their deriv | |
| $= 2\sqrt{3}$ | A1cao | | | | |
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| 3 | (i) | $y = e^{-x} \sin 2x$ $\Rightarrow \quad dy/dx = e^{-x} \cdot 2\cos 2x + (-e^{-x})\sin 2x$ | M1 B1 A1 [3] | Product rule $d/dx(\sin 2x) = 2 \cos 2x$ Any correct expression | $u \times \text{their } v' + v \times \text{their } u'$ but mark final answer |
| | (ii) | $dy/dx = 0$ when $2 \cos 2x - \sin 2x = 0$ $\Rightarrow \quad 2 = \tan 2x$ $\Rightarrow \quad 2x = \arctan 2$ $\Rightarrow \quad x = \frac{1}{2} \arctan 2^*$ | M1 M1 A1 [3] | ft their dy/dx but must eliminate e^{-x} $\sin 2x / \cos 2x = \tan 2x$ used [or \tan^{-1}] NB AG | derivative must have 2 terms substituting $\frac{1}{2} \arctan 2$ into their deriv M0 (unless $\cos 2x = 1/\sqrt{5}$ and $\sin 2x = 2/\sqrt{5}$ found) must show previous step |

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| 4 | (i) | translation in the x -direction of $\pi/4$ to the right translation in y -direction of 1 unit up. | M1 A1 M1 A1 [4] | allow 'shift', 'move' oe (eg using vector) allow 'shift', 'move' oe (eg using vector) | If just vectors given withhold one 'A' mark only 'Translate $\begin{pmatrix} \pi/4 \\ 1 \end{pmatrix}$ ' is 4 marks; if this is followed by an additional incorrect transformation, SC M1M1A1A0 $\begin{pmatrix} \pi/4 \\ 1 \end{pmatrix}$ only is M2A1A0 |
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| 4 (ii) | $g(x) = \frac{2 \sin x}{\sin x + \cos x}$ $g'(x) = \frac{(\sin x + \cos x)2 \cos x - 2 \sin x(\cos x - \sin x)}{(\sin x + \cos x)^2}$ $= \frac{2 \sin x \cos x + 2 \cos^2 x - 2 \sin x \cos x + 2 \sin^2 x}{(\sin x + \cos x)^2}$ $= \frac{2 \cos^2 x + 2 \sin^2 x}{(\sin x + \cos x)^2} = \frac{2(\cos^2 x + \sin^2 x)}{(\sin x + \cos x)^2}$ $= \frac{2}{(\sin x + \cos x)^2} *$ <p>When $x = \pi/4$, $g'(\pi/4) = 2/(1/\sqrt{2} + 1/\sqrt{2})^2$</p> $= 1$ $f'(x) = \sec^2 x$ $f'(0) = \sec^2(0) = 1, \text{ [so gradient the same here]}$ | <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[7]</p> | <p>Quotient (or product) rule consistent with their derivs</p> <p>Correct expanded expression (could leave the '2' as a factor)</p> <p>NB AG</p> <p>substituting $\pi/4$ into correct deriv</p> <p>o.e., e.g. $1/\cos^2 x$</p> | <p>(Can deal with num and denom separately)</p> <p>$\frac{vu' - uv'}{v^2}$; allow one slip, missing brackets</p> <p>$\frac{uv' - vu'}{v^2}$ is M0. Condone $\cos x^2$, $\sin x^2$</p> <p>must take out 2 as a factor or state $\sin^2 x + \cos^2 x = 1$</p> |

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| 4 (iii) | $\int_0^{\pi/4} f(x) dx = \int_0^{\pi/4} \frac{\sin x}{\cos x} dx$ <p>let $u = \cos x$, $du = -\sin x dx$ when $x = 0$, $u = 1$, when $x = \pi/4$, $u = 1/\sqrt{2}$</p> $= \int_1^{1/\sqrt{2}} -\frac{1}{u} du$ $= \int_{1/\sqrt{2}}^1 \frac{1}{u} du *$ $= [\ln u]_{1/\sqrt{2}}^1$ $= \ln 1 - \ln(1/\sqrt{2})$ $= \ln \sqrt{2} = \ln 2^{1/2} = \frac{1}{2} \ln 2$ | M1 A1 M1 A1 [4] | substituting to get $\int -1/u (du)$ NB AG $[\ln u]$ $\ln \sqrt{2}$, $\frac{1}{2} \ln 2$ or $-\ln(1/\sqrt{2})$ | ignore limits here, condone no du but not dx allow $\int 1/u \cdot -du$ but for A1 must deal correctly with the $-ve$ sign by interchanging limits mark final answer |
| (iv) | Area = area in part (iii) translated up 1 unit. So $= \frac{1}{2} \ln 2 + 1 \times \pi/4 = \frac{1}{2} \ln 2 + \pi/4$. | M1 A1cao [2] | soi from $\pi/4$ added oe (as above) | or $\int_{\pi/4}^{\pi/2} (1 + \tan(x - \pi/4)) dx = [x + \ln \sec(x - \pi/4)]_{\pi/4}^{\pi/2}$ $= \pi/2 + \ln \sqrt{2} - \pi/4 = \pi/4 + \ln \sqrt{2}$ B2 |

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| 5 | $y = x^2 \tan 2x$ $\Rightarrow dy/dx = 2x^2 \sec^2 2x + 2x \tan 2x$ OR $y = x^2 \frac{\sin 2x}{\cos 2x}$ $\frac{dy}{dx} = x^2 \frac{\cos 2x \cdot 2 \cos 2x - \sin 2x (-2 \sin 2x)}{\cos^2 2x} + 2x \frac{\sin 2x}{\cos 2x}$ $= \dots = 2x^2 \sec^2 2x + 2x \tan 2x$ OR $y = \frac{x^2 \sin 2x}{\cos 2x}$ $\frac{dy}{dx} = \frac{\cos 2x (2x \sin 2x + x^2 2 \cos 2x) - 2x^2 \sin 2x (-\sin 2x)}{\cos^2 2x}$ $= \dots = 2x^2 \sec^2 2x + 2x \tan 2x$ | M1 M1 A1cao M1 A1 A1cao M1 A1 A1cao [3] | product rule d/du(tan u) = sec ² u soi or 2x ² /cos ² 2x + 2xtan 2x product rule correct expression or 2x ² /cos ² 2x + 2xtan 2x (isw) quotient rule correct expression or 2x ² /cos ² 2x + 2xtan 2x (isw) | $u \times \text{their } v' + v \times \text{their } u'$ attempted M0 if d/dx (tan 2x) = (2) sec ² x isw <i>see additional notes for complete solution</i> $u \times \text{their } v' + v \times \text{their } u'$ attempted or (2x ² + 2xsin2xcos2x)/cos ² 2x or 2x ² /cos ² 2x + 2xsin2x / cos2x <i>see additional notes for complete solution</i> (v × their u' - u × their v')/v ² attempted or (2x ² + 2xsin2xcos2x)/cos ² 2x or 2x ² /cos ² 2x + 2xsin2x / cos2x |

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| 6 | $y = \sqrt[3]{1+x^2} = (1+x^2)^{1/3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{3}(1+x^2)^{-2/3} \cdot 2x$ $= \frac{2}{3}x(1+x^2)^{-2/3}$ | M1 M1 B1 A1 [4] | (1+x ²) ^{1/3} chain rule (1/3)u ^{-2/3} (soi) cao, mark final answer | Do not allow MR for square root their dy/du × du/dx (available for wrong indices) no ft on 1/2 index oe e.g. $\frac{2x(1+x^2)^{-2/3}}{3}$, $\frac{2x}{3\sqrt[3]{(1+x^2)^2}}$, etc but must combine 2 with 1/3. |
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| <p>7 $y = x^2(1 + 4x)^{1/2}$</p> <p>$\Rightarrow \frac{dy}{dx} = x^2 \cdot \frac{1}{2}(1 + 4x)^{-1/2} \cdot 4 + 2x(1 + 4x)^{1/2}$</p> <p>$= 2x(1 + 4x)^{-1/2}(x + 1 + 4x)$</p> <p>$= \frac{2x(5x + 1)}{\sqrt{1 + 4x}} *$</p> | <p>M1 B1 A1</p> <p>M1 A1 [5]</p> | <p>product rule with $u = x^2$, $v = \sqrt{1 + 4x}$ $\frac{1}{2}(\dots)^{-1/2}$ soi correct expression</p> <p>factorising or combining fractions NB AG</p> | <p>consistent with their derivatives; condone wrong index in v used for M1 only</p> <p>(need not factor out the $2x$) must have evidence of $x + 1 + 4x$ oe or $2x(5x + 1)(1 + 4x)^{-1/2}$ or $2x(5x + 1)/(1 + 4x)^{1/2}$</p> |
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