1 Fig. 8 shows part of the curve $y = x\cos 2x$, together with a point P at which the curve crosses the x-axis.

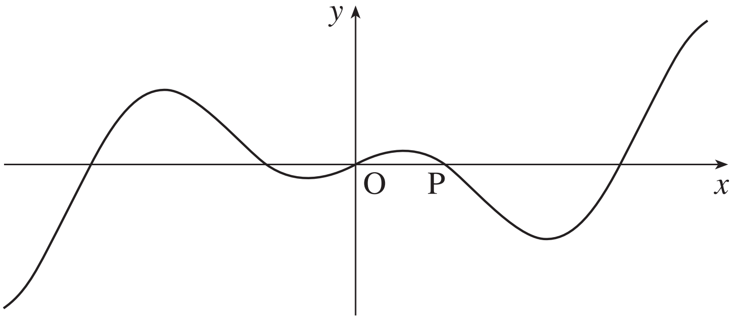

Fig. 8

(i) Find the exact coordinates of P. [3]

(ii) Show algebraically that $x\cos 2x$ is an odd function, and interpret this result graphically. [3]

(iii) Find $\dfrac{dy}{dx}$. [2]

(iv) Show that turning points occur on the curve for values of x which satisfy the equation $x\tan 2x = \frac{1}{2}$. [2]

(v) Find the gradient of the curve at the origin.

Show that the second derivative of $x\cos 2x$ is zero when $x = 0$. [4]

(vi) Evaluate $\int_0^{\frac{1}{4}\pi} x\cos 2x\,dx$, giving your answer in terms of π. Interpret this result graphically. [6]

2 Fig. 8 shows part of the curve $y = x \sin 3x$. It crosses the x-axis at P. The point on the curve with x-coordinate $\frac{1}{6}\pi$ is Q.

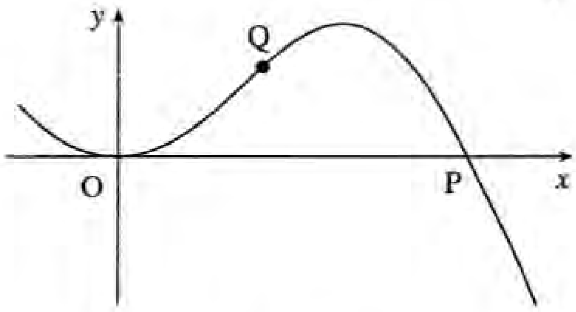

Fig. 8

(i) Find the x-coordinate of P. [3]

(ii) Show that Q lies on the line $y = x$. [1]

(iii) Differentiate $x \sin 3x$. Hence prove that the line $y = x$ touches the curve at Q. [6]

(iv) Show that the area of the region bounded by the curve and the line $y = x$ is $\frac{1}{72}(\pi^2 - 8)$. [7]

3 The function $f(x) = \ln(1 + x^2)$ has domain $-3 \leqslant x \leqslant 3$.

Fig. 9 shows the graph of $y = f(x)$.

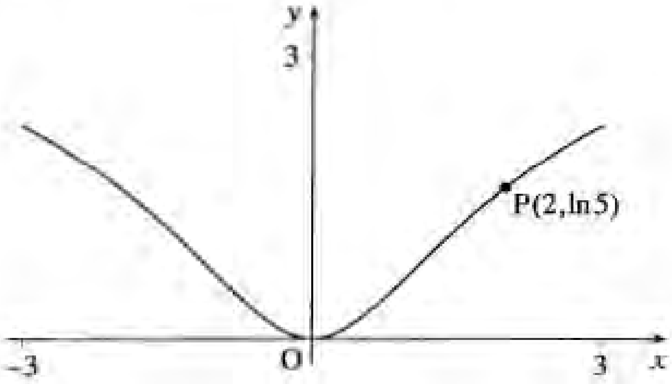

Fig. 9

(i) Show algebraically that the function is even. State how this property relates to the shape of the curve. [3]

(ii) Find the gradient of the curve at the point P$(2, \ln 5)$. [4]

(iii) Explain why the function does not have an inverse for the domain $-3 \leqslant x \leqslant 3$. [1]

The domain of $f(x)$ is now restricted to $0 \leqslant x \leqslant 3$. The inverse of $f(x)$ is the function $g(x)$.

(iv) Sketch the curves $y = f(x)$ and $y = g(x)$ on the same axes.

State the domain of the function $g(x)$.

Show that $g(x) = \sqrt{e^x - 1}$. [6]

(v) Differentiate $g(x)$. Hence verify that $g'(\ln 5) = 1\frac{1}{4}$. Explain the connection between this result and your answer to part **(ii)**. [5]