1 Fig. 8 shows the curve $y = 3\ln x + x - x^2$.

The curve crosses the x-axis at P and Q, and has a turning point at R. The x-coordinate of Q is approximately 2.05.

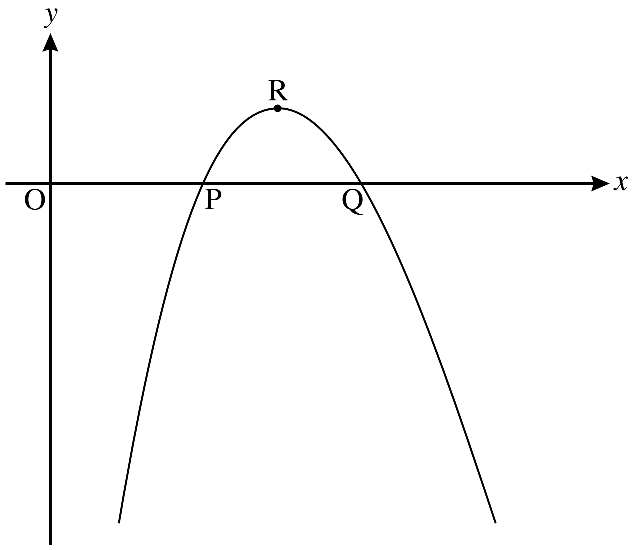

Fig. 8

(i) Verify that the coordinates of P are (1, 0). **[1]**

(ii) Find the coordinates of R, giving the y-coordinate correct to 3 significant figures.

Find $\frac{d^2y}{dx^2}$, and use this to verify that R is a maximum point. **[9]**

(iii) Find $\int \ln x \, dx$.

Hence calculate the area of the region enclosed by the curve and the x-axis between P and Q, giving your answer to 2 significant figures. **[7]**

2 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{\mathrm{e}^{2x}}{1 + \mathrm{e}^{2x}}$. The curve crosses the y-axis at P.

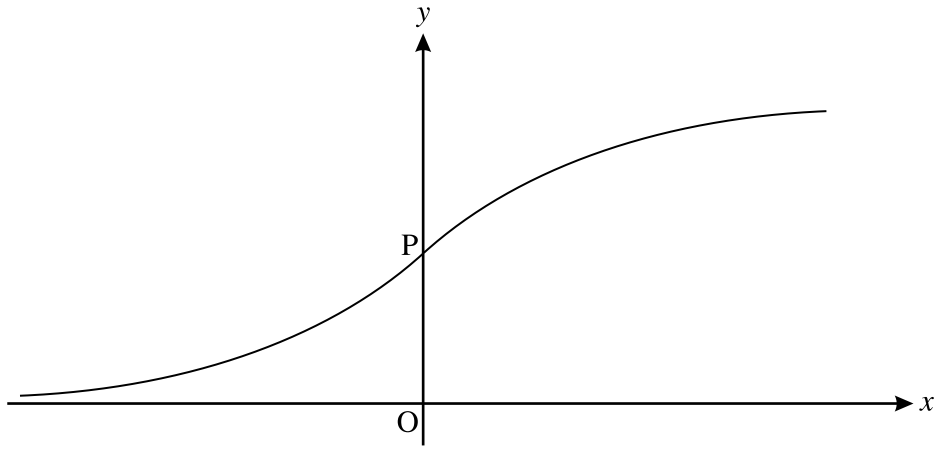

Fig. 9

(i) Find the coordinates of P. **[1]**

(ii) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, simplifying your answer.

Hence calculate the gradient of the curve at P. **[4]**

(iii) Show that the area of the region enclosed by $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = 1$ is $\frac{1}{2}\ln\left(\dfrac{1 + \mathrm{e}^2}{2}\right)$. **[5]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \frac{1}{2}\left(\dfrac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}\right)$.

(iv) Prove algebraically that $\mathrm{g}(x)$ is an odd function.

Interpret this result graphically. **[3]**

(v) (A) Show that $\mathrm{g}(x) + \frac{1}{2} = \mathrm{f}(x)$.

(B) Describe the transformation which maps the curve $y = \mathrm{g}(x)$ onto the curve $y = \mathrm{f}(x)$.

(C) What can you conclude about the symmetry of the curve $y = \mathrm{f}(x)$? **[6]**

3 A curve is defined by the equation $y = 2x\ln(1+x)$.

(i) Find $\dfrac{dy}{dx}$ and hence verify that the origin is a stationary point of the curve. [4]

(ii) Find $\dfrac{d^2y}{dx^2}$, and use this to verify that the origin is a minimum point. [5]

(iii) Using the substitution $u = 1 + x$, show that $\displaystyle\int \frac{x^2}{1+x}\,dx = \int \left(u - 2 + \frac{1}{u}\right) du$.

Hence evaluate $\displaystyle\int_0^1 \frac{x^2}{1+x}\,dx$, giving your answer in an exact form. [6]

(iv) Using integration by parts and your answer to part **(iii)**, evaluate $\displaystyle\int_0^1 2x\ln(1+x)\,dx$. [4]