1(i) When $x = 1$, $y = 3 \ln 1 + \quad {}^{2}$ $= 0$	E1 [1]		
(ii) $\frac{dy}{dx} = \frac{3}{x} + 1 - 2x$ $\Rightarrow$ At R, $\frac{dy}{dx} = 0 = \frac{3}{x} + 1 - 2x$ $\Rightarrow$ $3 + x - 2x^2 = 0$ $\Rightarrow$ $(3 - 2x)(1 + x) = 0$ $\Rightarrow$ $x = 1.5$, (or -1) $\Rightarrow$ $y = 3 \ln 1.5 + 1.5 - 1.5^2$ $= 0.466$ (3 s.f.) $\frac{d^2y}{dx^2} = -\frac{3}{x^2} - 2$ When $x = 1.5$, d^2y/dx^2 ($= -10/3$) $< 0 \Rightarrow$ max	M1 A1cao M1 M1 A1 M1 A1cao B1ft E1 [9]	d/dx (ln x) = $1/x$ re-arranging into a quadratic = 0 factorising or formula or completing square substituting their x ft their dy/dx on equivalent work www – don't need to calculate 10/3	 SC1 for $x = 1.5$ unsupported, SC3 if verified but condone rounding errors on 0.466
(iii) Let $u = \ln x$, $du/dx = 1/x$ $dv/dx = 1$, $v = x$ $\Rightarrow$ $\int \ln x dx = x \ln x - \int x.\frac{1}{x} dx$ $= x \ln x - \int 1.dx$ $= x \ln x - x + c$ $\Rightarrow$ $A = \int_1^{2.05} (3 \ln x + x - x^2) dx$ $= \left[3x \ln x - 3x + \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_1^{2.05}$ $= -2.5057 + 2.833..$ $= 0.33$ (2 s.f.)	 M1 A1 A1 B1 B1ft M1dep A1 cao [7]	 parts condone no c correct integral and limits (soi) $\left[3 \times \text{their } 'x \ln x - x' + \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]$ substituting correct limits dep 1st B1	 allow correct result to be quoted (SC3)

2(i) (0, ½)	B1 [1]	allow y = ½ , but not (x =) ½ or (½ ,0) nor P = 1/2	
(ii) $\frac{dy}{dx}=\frac{(1+e^{2x})2e^{2x}-e^{2x}.2e^{2x}}{(1+e^{2x})^2}$ $=\frac{2e^{2x}}{(1+e^{2x})^2}$ When x = 0, $dy/dx = 2e^0/(1+e^0)^2$ = ½	M1 A1 A1 B1ft [4]	Quotient or product rule correct expression – condone missing bracket cao – mark final answer follow through their derivative	product rule: $\frac{dy}{dx}=e^{2x}.2e^{2x}(-1)(1+e^{2x})^{-2}+2e^{2x}(1+e^{2x})^{-1}$ $-\frac{2e^{2x}}{(1+e^{2x})^2}$ from $(udv - vdu)/v^2$ SC1
(iii) $A=\int_0^1 \frac{e^{2x}}{1+e^{2x}}dx$ $=\left[\frac{1}{2}\ln(1+e^{2x})\right]_0^1$ *or* let $u = 1 + e^{2x}$, $du/dx = 2e^{2x}$ $\Rightarrow$ $A=\int_2^{1+e^2}\frac{1/2}{u}du = \left[\frac{1}{2}\ln u\right]_2^{1+e^2}$ $=\frac{1}{2}\ln(1+e^2)-\frac{1}{2}\ln 2$ $=\frac{1}{2}\ln\left[\frac{1+e^2}{2}\right]$*	B1 M1 A1 M1 A1 M1 E1 [5]	correct integral and limits (soi) $k\ln(1+e^{2x})$ k = ½ or $v = e^{2x}$, $dv/dx = 2e^{2x}$ o.e. [½ ln u] or [½ ln (v + 1)] substituting correct limits www	condone no dx allow missing dx's or incompatible limits, but penalise missing brackets
(iv) $g(-x)=\frac{1}{2}\left[\frac{e^{-x}-e^{x}}{e^{-x}+e^{x}}\right]=-\frac{1}{2}\left[\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\right]=-g(x)$ Rotational symmetry of order 2 about O	M1 E1 B1 [3]	substituting $-x$ for x in g(x) completion www – taking out –ve must be clear must have 'rotational' 'about O', 'order 2' (oe)	not g($-x$) ≠ g(x). Condone use of f for g.
(v)(*A*) $g(x)+\frac{1}{2}=\frac{1}{2}.\frac{e^x-e^{-x}}{e^x+e^{-x}}+\frac{1}{2}=\frac{1}{2}.(\frac{e^x-e^{-x}+e^x+e^{-x}}{e^x+e^{-x}})$ $=\frac{1}{2}.(\frac{2e^x}{e^x+e^{-x}})$ $=\frac{e^x.e^x}{e^x(e^x+e^{-x})}=\frac{e^{2x}}{e^{2x}+1}=f(x)$ (*B*) ranslation $\begin{pmatrix}0\\1/2\end{pmatrix}$ (*C*) Rotational symmetry [of order 2]about P	M1 A1 E1 M1 A1 B1 [6]	combining fractions (correctly) translation in y direction up ½ unit dep 'translation' used o.e. condone omission of 180°/order 2	allow 'shift', 'move' in correct direction for M1. $\begin{pmatrix}0\\1/2\end{pmatrix}$ alone is SC1.

3(i) $y = 2x\ln(1+x)$ $\Rightarrow \quad \frac{dy}{dx} = \frac{2x}{1+x} + 2\ln(1+x)$ When $x = 0$, $dy/dx = 0 + 2\ln 1 = 0$ $\Rightarrow$ origin is a stationary point.	M1 B1 A1 E1 [4]	product rule $d/dx(\ln(1+x)) = 1/(1+x)$ soi www (i.e. from correct derivative)
(ii) $\frac{d^2y}{dx^2} = \frac{(1+x).2 - 2x.1}{(1+x)^2} + \frac{2}{1+x}$ $= \frac{2}{(1+x)^2} + \frac{2}{1+x}$ When $x = 0$, $d^2y/dx^2 = 2 + 2 = 4 > 0$ $\Rightarrow$ (0, 0) is a min point	M1 A1ft A1 M1 E1 [5]	Quotient or product rule on their $2x/(1+x)$ correctly applied to their $2x/(1+x)$ o.e., e.g. $\frac{4+2x}{(1+x)^2}$ cao substituting $x = 0$ into their d^2y/dx^2 www – dep previous A1
(iii) Let $u = 1 + x \Rightarrow du = dx$ $\Rightarrow \int \frac{x^2}{1+x}dx = \int \frac{(u-1)^2}{u}du$ $= \int \frac{(u^2-2u+1)}{u}du$ $= \int (u - 2 + \frac{1}{u})du$ * $\Rightarrow \int_0^1 \frac{x^2}{1+x}dx = \int_1^2 (u-2+\frac{1}{u})du$ $= \left[\frac{1}{2}u^2 - 2u + \ln u\right]_1^2$ $= 2 - 4 + \ln 2 - (½ - 2 + \ln 1)$ $= \ln 2 - ½$	M1 E1 B1 B1 M1 A1 [6]	$\frac{(u-1)^2}{u}$ www (but condone du omitted except in final answer) changing limits (or substituting back for x and using 0 and 1) $\left[\frac{1}{2}u^2 - 2u + \ln u\right]$ substituting limits (consistent with u or x) cao
(iv) $A = \int_0^1 2x\ln(1+x)dx$ Parts: $u = \ln(1+x)$, $du/dx = 1/(1+x)$ $dv/dx = 2x \Rightarrow v = x^2$ $= \left[x^2\ln(1+x)\right]_0^1 - \int_0^1 \frac{x^2}{1+x}dx$ $= \ln 2 - \ln 2 + ½ = ½$	M1 A1 M1 A1 [4]	soi substituting their $\ln 2 - ½$ for $\int_0^1 \frac{x^2}{1+x}dx$ cao