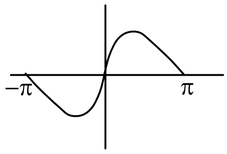
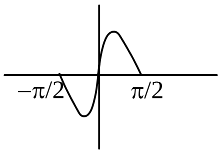
1(i) $a = 1/3$	B1 [1]	or 0.33 or better
(ii) $\frac{dy}{dx} = \frac{(3x-1)2x - x^2.3}{(3x-1)^2}$ $= \frac{6x^2 - 2x - 3x^2}{(3x-1)^2}$ $= \frac{3x^2 - 2x}{(3x-1)^2}$ $= \frac{x(3x-2)}{(3x-1)^2}$ *	M1 A1 E1 [3]	quotient rule www – must show both steps; penalise missing brackets.
(iii) dy/dx = 0 when $x(3x – 2) = 0$ $\Rightarrow$ $x = 0$ or $x = 2/3$, so at P, $x = 2/3$ when $x = \frac{2}{3}$, $y = \frac{(2/3)^2}{3\times(2/3)-1} = \frac{4}{9}$ when $x = 0.6$, dy/dx = –0.1875 when $x = 0.8$, dy/dx = 0.1633 Gradient increasing $\Rightarrow$ minimum	M1 A1 M1 A1cao B1 B1 E1 [7]	if denom = 0 also then M0 o.e e.g. $0.\dot{6}$, but must be exact o.e e.g. $0.\dot{4}$, but must be exact –3/16, or –0.19 or better 8/49 or 0.16 or better o.e. e.g. ‘from negative to positive’. Allow ft on their gradients, provided –ve and +ve respectively. Accept table with indications of signs of gradient.
(iv) $\int \frac{x^2}{3x-1}dx$ $u = 3x - 1 \Rightarrow du = 3dx$ $= \int \frac{\frac{(u+1)^2}{9}}{u}\frac{1}{3}du$ $= \frac{1}{27}\int\frac{(u+1)^2}{u}du = \frac{1}{27}\int\frac{u^2+2u+1}{u}du$ $= \frac{1}{27}\int(u+2+\frac{1}{u})du$ * Area = $\int_{2/3}^{1}\frac{x^2}{3x-1}dx$ When $x = 2/3$, $u =1$, when $x = 1$, $u = 2$ $= \frac{1}{27}\int_1^2(u+2+1/u)du$ $= \frac{1}{27}\left[\frac{1}{2}u^2 + 2u + \ln u\right]_1^2$ $= \frac{1}{27}[(2+4+\ln 2) - (\frac{1}{2}+2+\ln 1)]$ $= \frac{1}{27}(3\frac{1}{2} + \ln 2)$ $[= \frac{7+2\ln 2}{54}$]	 B1 M1 M1 E1 B1 M1 A1cao [7]	 $\frac{\frac{(u+1)^2}{9}}{u}$ o. × 1/3 (du) expanding Condone missing du’s $\left[\frac{1}{2}u^2 + 2u + \ln u\right]$ substituting correct limits, dep integration o.e., but must evaluate ln 1 = 0 and collect terms.

2 $y=(1+6x^2)^{1/3}$ $\Rightarrow \frac{dy}{dx}=\frac{1}{3}(1+6x^2)^{-2/3}.12x$ $=4x(1+6x^2)^{-2/3}$	M1 B1 A1 A1 [4]	chain rule used $\frac{1}{3}u^{-2/3}$ ×12x cao (must resolve 1/3 × 12) Mark final answer

3 $y = x^2\ln x$ $\Rightarrow \frac{dy}{dx}=x^2.\frac{1}{x}+2x\ln x$ $=x+2x\ln x$ dy/dx = 0 when x +2$x$ln x = 0 $\Rightarrow$ $x(1 + 2\ln x) = 0$ $\Rightarrow$ $\ln x = -½$ $\Rightarrow$ $x = e^{-½} = 1/\sqrt{e}$ *	M1 B1 A1 M1 M1 E1 [6]	product rule d/dx (ln x) = 1/x soi oe their deriv = 0 or attempt to verify $\ln x = -½ \Rightarrow x = e^{-½}$ or ln (1/√e) = – ½

4 (i) $y=\frac{x^2}{2x+1}$ $\Rightarrow \frac{dy}{dx}=\frac{(2x+1)2x-x^2.2}{(2x+1)^2}$ $=\frac{2x^2+2x}{(2x+1)^2}=\frac{2x(x+1)}{(2x+1)^2}$ *	M1 A1 A1 E1 [4]	Use of quotient rule (or product rule) Correct numerator – condone missing bracket provided it is treated as present Correct denominator www –do not condone missing brackets
(ii) $\frac{dy}{dx}=0$ when $2x(x + 1) = 0$ $\Rightarrow$ $x = 0$ or -1 $y = 0$ or -1	B1 B1 B1 B1 [4]	Must be from correct working: SC −1 if denominator = 0

5 (i) ½ $(1 + 2x)^{-1/2} \times 2$ $= \dfrac{1}{\sqrt{1+2x}}$	M1 B1 A1 [3]	chain rule ½ $u^{-1/2}$ or ½ $(1 + 2x)^{-1/2}$ oe, but must resolve ½ × 2 = 1
(ii) $y = \ln(1 - e^{-x})$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{1-e^{-x}}.(-e^{-x})(-1)$ $= \dfrac{e^{-x}}{1-e^{-x}}$ $= \dfrac{1}{e^{x}-1}$ *	M1 B1 A1 E1 [4]	chain rule $\dfrac{1}{1-e^{-x}}$ or $\dfrac{1}{u}$ if substituting $u = 1 - e^{-x}$ $\times(-e^{-x})(-1)$ or e^{-x} www (may imply $\times e^{x}$ top and bottom)

6 (i) $f(-x) = \dfrac{\sin(-x)}{2-\cos(-x)}$ $= \dfrac{-\sin(x)}{2-\cos(x)}$ $= -f(x)$	M1 A1	substituting $-x$ for x in $f(x)$
	B1 [3]	Graph completed with rotational symmetry about O.
(ii) $f'(x) = \dfrac{(2-\cos x)\cos x - \sin x.\sin x}{(2-\cos x)^2}$	M1	Quotient or product rule consistent with their derivatives
$= \dfrac{2\cos x - \cos^2 x - \sin^2 x}{(2-\cos x)^2}$	A1	Correct expression
$= \dfrac{2\cos x - 1}{(2-\cos x)^2}$ *	E1	
$f'(x) = 0$ when $2\cos x - 1 = 0$ $\Rightarrow \cos x = ½$, $x = \pi/3$	M1 A1	numerator = 0
When $x = \pi/3$, $y = \dfrac{\sin(\pi/3)}{2-\cos(\pi/3)} = \dfrac{\sqrt{3}/2}{2-1/2}$	M1	Substituting their $\pi/3$ into y
$= \dfrac{\sqrt{3}}{3}$	A1	o.e. but exact
So range is $-\dfrac{\sqrt{3}}{3} \le y \le \dfrac{\sqrt{3}}{3}$	B1ft [8]	ft their $\dfrac{\sqrt{3}}{3}$
(iii) $\int_0^{\pi} \dfrac{\sin x}{2-\cos x}dx$ let $u = 2 - \cos x$ $\Rightarrow du/dx = \sin x$	M1	$\int \dfrac{1}{u}du$
When $x = 0$, $u = 1$; when $x = \pi$, $u = 3$ $= \int_1^3 \dfrac{1}{u}du$	B1	$u = 1$ to 3
$= [\ln u]_1^3$	A1ft	$[\ln u]$
$= \ln 3 - \ln 1 = \ln 3$	A1cao	
or $= [\ln(2-\cos x)]_0^{\pi}$ $= \ln 3 - \ln 1 = \ln 3$	M2 A1 A1 cao [4]	$[k \ln(2 - \cos x)]$ $k = 1$
(iv) 	B1ft [1]	Graph showing evidence of stretch s f. ½ in x – direction
(v) Area is stretched with scale factor ½ So area is ½ ln 3	M1 A1ft [2]	soi ½ their ln 3

7 (i) $x - 1 = \sin y$ $\Rightarrow$ $x = 1 + \sin y$ $\Rightarrow$ $dx/dy = \cos y$	M1 A1 E1	 www
(ii) When $x = 1.5$, $y = \arcsin(0.5) = \pi/6$ $\frac{dy}{dx} = \frac{1}{\cos y}$ $= \frac{1}{\cos \pi / 6}$ $= 2/\sqrt{3}$	M1 A1 M1 A1 [7]	condone 30° or 0.52 or better or $\frac{dy}{dx} = \frac{1}{\sqrt{1-(x-1)^2}}$ or equivalent, but must be exact

8 $y = \frac{x}{2+3\ln x}$ $\Rightarrow$ $\frac{dy}{dx} = \frac{(2+3\ln x).1 - x.\frac{3}{x}}{(2+3\ln x)^2}$ $= \frac{2+3\ln x - 3}{(2+3\ln x)^2}$ $= \frac{3\ln x - 1}{(2+3\ln x)^2}$	M1 B1 A1	Quotient rule consistent with their derivatives or product rule + chain rule on $(2+3x)^{-1}$ $\frac{d}{dx}(\ln x) = \frac{1}{x}$ soi correct expression
When $\frac{dy}{dx} = 0$, $3\ln x - 1 = 0$ $\Rightarrow$ $\ln x = 1/3$ $\Rightarrow$ $x = e^{1/3}$ $\Rightarrow$ $y = \frac{e^{1/3}}{2+1} = \frac{1}{3}e^{1/3}$	M1 A1cao M1 A1cao [7]	their numerator = 0 (or equivalent step from product rule formulation) M0 if denominator = 0 is pursued $x = e^{1/3}$ substituting for their x (correctly) Must be exact: – 0.46… is M1A0