1

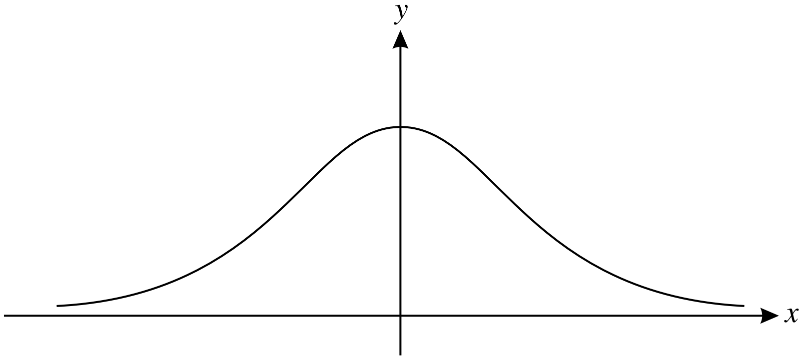

Fig. 8

Fig. 8 shows the curve $y = f(x)$, where $f(x) = \dfrac{1}{e^x + e^{-x} + 2}$.

(i) Show algebraically that $f(x)$ is an even function, and state how this property relates to the curve $y = f(x)$. **[3]**

(ii) Find $f'(x)$. **[3]**

(iii) Show that $f(x) = \dfrac{e^x}{(e^x + 1)^2}$. **[2]**

(iv) Hence, using the substitution $u = e^x + 1$, or otherwise, find the exact area enclosed by the curve $y = f(x)$, the x-axis, and the lines $x = 0$ and $x = 1$. **[5]**

(v) Show that there is only one point of intersection of the curves $y = f(x)$ and $y = \frac{1}{4}e^x$, and find its coordinates. **[5]**

2 Evaluate $\displaystyle\int_0^{\frac{1}{6}\pi} \cos 3x \, dx$. **[3]**

3 **(i)** Differentiate $x\cos 2x$ with respect to x. **[3]**

(ii) Integrate $x\cos 2x$ with respect to x. **[4]**

4 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{\sqrt{2x - x^2}}$.

The curve has asymptotes $x = 0$ and $x = a$.

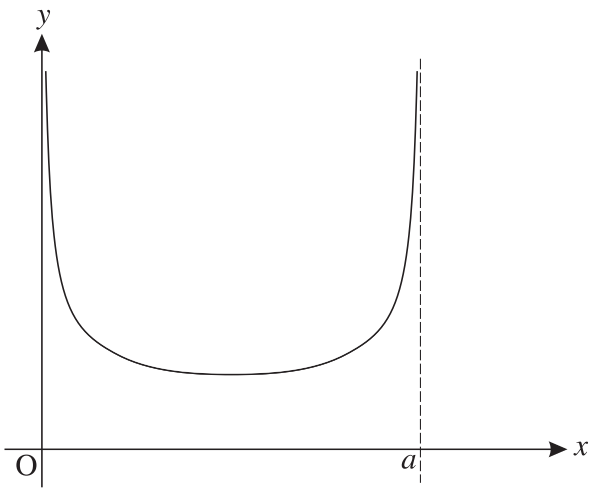

Fig. 9

(i) Find a. Hence write down the domain of the function. **[3]**

(ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x - 1}{\left(2x - x^2\right)^{\frac{3}{2}}}$.

Hence find the coordinates of the turning point of the curve, and write down the range of the function. **[8]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \dfrac{1}{\sqrt{1 - x^2}}$.

(iii) (A) Show algebraically that $\mathrm{g}(x)$ is an even function.

(B) Show that $\mathrm{g}(x - 1) = \mathrm{f}(x)$.

(C) Hence prove that the curve $y = \mathrm{f}(x)$ is symmetrical, and state its line of symmetry. **[7]**